Appendix (by the Editor): On Relativistic Mass

"Mass is a mess" [1]. Not quite...

During the last three decades physicists have witnessed (or rather endured) "what has probably been the most vigorous campaign ever waged against the concept of relativistic mass"[1] [2].

It seems that campaign had been prompted by Adler's paper "Does mass really depend on velocity, dad?" [3] in which he had even discovered support for his denial of the relativistic mass in Einstein's view on this concept [3, p. 742]:

> Whatever Einstein's precise early views were on the subject, his view in later life appears clear. In a 1948 letter to Lincoln Barnett, he wrote
>
> "It is not good to introduce the concept of the mass $M = m/(1 - v^2/c^2)^{1/2}$ of a body for which no clear definition can be given. It is better to introduce no other mass than the 'rest mass' m. Instead of introducing M, it is better to mention the expression for the momentum and energy of a body in motion."

Unfortunately, Einstein's unclear view of the relativistic mass[2] appears to have provided some encouragement for the campaign against the use of relativistic mass, but the above quote does not in fact demonstrate that "his view in later life appears clear" – Einstein merely expresses his concern and reservation about the definition of M; this becomes evident when it is taken into account that the translation of the above part of Einstein's letter is inaccurate and misleading – compare the translation by Ruschin (there are no such phrases as "introduce no other mass than" and "Instead of introducing M") [7] :

> The German word *daneben* does not mean "instead of," but rather "besides," "in addition to" or "moreover." I would therefore translate the passage:

[1] For a detailed account of the controversy over relativistic mass see Chapter 2 of Max Jammer's excellent book *Concepts of Mass in Contemporary Physics and Philosophy* [2].

[2] In his 1905 paper [4] Einstein defined two relativistic masses – longitudinal and transverse masses – but later avoided the entire concept of relativistic mass, including in this book. See [5], [6].

> It is not proper to speak of the mass $M = m/(1 - v^2/c^2)^{1/2}$ of a moving body, because no clear definition can be given for M. It is preferable to restrict oneself to the "rest mass" m. Besides, one may well use the expression for momentum and energy when referring to the inertial behavior of rapidly moving bodies.[3]

Two years after Adler's paper L. B. Okun started a series of publications [8]-[14], which seem to had been the driving force behind the unprecedented campaign against the concept of relativistic mass. In May 1990 *Physics Today* published a number of letters to the Editor with comments on Okun's first article [8] and Okun's replies. W. Rindler's reaction was the sharpest [15]:

> I am disturbed by the harm that Lev Okun's earnest tirade (June 1989 page 31) against the use of relativistic mass ("It is our duty... to stop this process") might do to the teaching of relativity. It might suggest to some who have not thought these matters through that there are unresolved logical difficulties in elementary relativity or that if they use the quantity $m = \gamma\, m_0$ they commit some physical blunder, whereas in fact this entire ado is about terminology.

Unfortunately, after that exchange "Okun's polemic condemnation" [2, p. 53] even escalated – here are just two examples of his choice of words: "The pedagogical virus of relativistic mass" (from the abstract of a paper [11]) and "The Virus of Relativistic Mass in the Year of Physics" (a title of a paper published in the volume [12]). I think it is truly sad that such a prominent particle physicist did not seem to have even attempted to entertain the possibility that he might have been fundamentally wrong.

While I share the feeling behind Rindler's reaction, I tend to disagree that "this entire ado is about terminology". And papers in support of the relativistic mass do show that the controversy implies more than terminology (see, for example, [16],[17]); here is the conclusion of Bickerstaff and Patsakos' paper [17, p. 66]:

> Thus we conclude by noting that in answering the elementary question of why two different masses are allowed in relativity, one obtains a clearer picture of the subject—a picture that is rooted in mathematics and logic rather than semantics and opinion.

Some physicists have argued that "There is no really good definition of mass" [18]-[21], which might explain the relativistic mass controversy. I tend to disagree with this too. The accepted[4] definition of mass – *the*

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[3] A scan of Einstein's letter in German is included in Okun's article [8].

[4] Only several examples: "Mass is that property of an object that specifies how much resistance an object exhibits to changes in its velocity" [22]; "mass [is] the resistance of a body to a change of motion" [23]; Mass is "the quantitative or numerical measure of a

mass of a particle is the measure of the resistance the particle offers to its acceleration – is both adequate for the concept of mass in relativity and does *indisputably* demonstrate that mass indeed increases with velocity and therefore relativistic mass is an integral part of relativity (complementing proper or rest mass):[5]

- a particle whose velocity increases and approaches the velocity of light *offers an increasing resistance to its acceleration*, that is, *obviously*, its mass (the measure of the resistance the particle offers to its acceleration) increases.[6]
- as in relativity the acceleration of a particle is different in different reference frames, the particle's mass is not an invariant since it is not the same in all frames (only the proper or rest mass, measured in the frame in which the particle is at rest, is an invariant).

These facts make the campaign against the concept of relativistic mass both inexplicable and worrisome. Instead of initiating and stimulating research on the origin of relativistic mass (and on the nature of mass in general) in order to achieve a more profound understanding of this fundamental concept in physics,[7] the relativistic mass is not mentioned at all in

body's inertia, that is of its resistance to being accelerated" [24]; "We use the term *mass* as a quantitative measure of inertia" [25, p. 9-1]; "Mass... measures how hard we have to push a body to achieve a given acceleration" [26]; "Mass is a quantitative measure of inertia... the greater its mass, the more a body "resists" being accelerated" [27]; "*The qualitative definition of the (inertial) mass of a particle is that it is a numerical measure of the reluctance of the particle to being accelerated*" [28]; "mass is a measure of the inertia of an object" [29]; Mass is defined as the "resistance to acceleration" [30]; "Mass is the measure of the gravitational and inertial properties of matter" [31].

[5] As I think it is exceedingly obvious that there are two masses in relativity (like two times; see below) – rest (or proper) mass and relativistic mass – I do not see any need to comment on the problems (coming from the equivalence of mass and energy) with a *single* concept of mass in relativity ("as an invariant, intrinsic property of an object" [32]).

[6] Arguing, effectively, that not the particle's mass but the particle's inertia increases [33] amounts to a rejection of the accepted definition of mass without a valid reason; inertia is the *phenomenon* of offering resistance to acceleration, whereas mass is the *measure* of that resistance.

[7] More research is needed to address the obvious situation: As the *resistance* of a particle to its acceleration depends on the acceleration's direction (the resistance is greater when the acceleration is along the particle's velocity and is becoming infinite as the particle's velocity is approaching the velocity of light), its mass is rather a tensor, not a scalar. In his 1905 paper [4] Einstein defined the two relativistic masses – longitudinal and transverse masses – but later silently abandoned them. With respect to the relativistic masses (longitudinal and transverse) we may witness a repetition of the story with the cosmological constant – initially Einstein used the cosmological constant in his equation linking matter and energy with the spacetime curvature, but later he called it the "biggest blunder of my life;" now cosmologists reintroduced Einstein's cosmological constant. At present time the relativistic mass (let alone the longitudinal and transverse masses) is so out of fashion that even such a prominent relativist as Wolfgang Rindler had to choose the words "confess" and "heuristic" in his letter to the Editor of *Physics Today* [15]: "I will confess to even occasionally using the heuristic concepts of longitudinal mass $\gamma^3 m_0$ and transverse mass γm_0 to predict how a particle will move in a given field of force."

many publications[8] (see, for example, the well-known textbook [35]) or, if it is mentioned, it is done to caution the readers[9], that "Most physicists prefer to consider the mass of a particle as fixed" [25, p. 760], that "Most physicists prefer to keep the concept of mass as an invariant, intrinsic property of an object" [32], that "We choose not to use relativistic mass, because it can be a misleading concept" [36] or to warn them [22, p. 1215]:

> **Watch Out for "Relativistic Mass"**
> Some older treatments of relativity maintained the conservation of momentum principle at high speeds by using a model in which a particle's mass increases with speed. You might still encounter this notion of "relativistic mass" in your outside reading, especially in older books. Be aware that this notion is no longer widely accepted; today, mass is considered as *invariant*, independent of speed. The mass of an object in all frames is considered to be the mass as measured by an observer at rest with respect to the object.

But phrases such as "*prefer* to consider," "*prefer* to keep," "*choose* not to use" (and "*can be*"), "*no longer widely accepted*" and even "older treatments" do not belong to the rigorous language of physics. Physics is not fashion where expressions such as "*prefer to*" and "*choose not to use*," for example, naturally fit. Physics at its best asks and addresses questions such as:

- Why is the velocity of light the greatest velocity, which cannot be reached by a particle possessing rest mass?
- Why does such a particle offer an *increasing resistance*[10] as its velocity increases and approaches the velocity of light? Or, which is the same question, why does the mass of a particle increase as its velocity increases and approaches the velocity of light?

I think, it is evident from here that the relativistic increase of mass is an *experimental fact* – it is an experimental fact that a particle offers an *increasing resistance* as its velocity increases and approaches the velocity of light and its mass is the measure of that resistance. Therefore the relativistic mass reflects the experimental evidence and we are not free to decide whether or not to use it.

[8] For a list of published works using relativistic mass see [34]. Here I think it is worth mentioning specifically Feynman: "Mass is found to increase with velocity, but appreciable increases require velocities near that of light" [25].

[9] Some authors prefer to take a neutral position: "The use of relativistic mass has its supporters and detractors, some quite strong in their opinions. We will mostly deal with individual particles, so we will sidestep the controversy and use Eq. (37.27) [$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}\,(1-v^2/c^2)^{-1/2}$] as the generalized definition of momentum with m as a constant for each particle, independent of its state of motion" [27, p. 1244]

[10] In fact, the profound question of the nature of inertia and mass (i.e., the question of the *origin* of the *resistance* a particle offers to its acceleration) has been an open one since Galileo and Newton [37]. The discovery that mass increases with velocity and the controversy over relativistic mass made the need to try to address this open question more urgent.

Finally, here I will summarize my response [39] to the objections of Taylor and Wheeler [40] against using the concept of relativistic mass. Here are their objections:

> The concept of 'relativistic mass' is subject to misunderstanding [...]. First, it applies the name mass – belonging to the magnitude of a 4-vector – to a very different concept, the time component of a 4-vector. Second, it makes increase of energy of an object with velocity or momentum appear to be connected with some change in internal structure of the object. In reality, the increase of energy with velocity originates not in the object but in the geometric properties of spacetime itself.

It is true that the magnitude of the four-momentum is proportional to the rest mass of a particle:

$$|\vec{p}| = mc \ .$$

The time component of the four-momentum

$$p^0 = \frac{mc}{(1 - v^2/c^2)^{1/2}} = m(v)c$$

is proportional to the relativistic mass $m(v) = m(1-v^2/c^2)^{-1/2}$. So the rest (proper) mass m is indeed proportional to the magnitude of a four-vector and is an invariant, whereas the relativistic mass $m(v)$ is a component of a four-vector.

However, the situation is precisely the same with respect to proper time and coordinate time. The square of the spacetime distance Δs^2 between two events lying on a timelike worldline is equal to the scalar product $\Delta\vec{x}\cdot\Delta\vec{x}$ of the displacement four-vector $\Delta\vec{x}$ connecting the two events. In other words, the magnitude of the displacement vector is equal to the spacetime distance along the timelike worldline:

$$|\Delta\vec{x}| = \Delta s \ .$$

As $\Delta s = c\Delta\tau$, the magnitude of $\Delta\vec{x}$ is proportional to the proper time $\Delta\tau$ between the two events on the timelike worldline that are connected by the displacement vector:

$$|\Delta\vec{x}| = c\Delta\tau \ .$$

Therefore, the magnitude of the four-vector $\Delta\vec{x}$ is proportional to the proper time $\Delta\tau$.

On the other hand, however, coordinate time is the zeroth (time) component $\Delta x^0 = c\Delta t$ of the displacement four-vector $\Delta\vec{x}$.

So, if we cannot talk about relativistic mass, by the same argument we should talk only about proper time, which is an invariant, and deny the name 'time' to the coordinate time; however, it is the coordinate time that changes relativistically – the experimentally tested time dilation involves precisely coordinate time.

Therefore, proper or rest mass (which is an invariant) and relativistic mass (which is frame-dependent) are exactly like proper time (which is an invariant) and relativistic / coordinate time (which is frame-dependent) [and, to some extent, like proper and relativistic length].

As we saw above, this becomes even more evident from the very definition of mass as the measure of the *resistance* a particle offers to its acceleration or, in the framework of relativity, as the measure of the resistance a particle offers when deviated from its geodesic path. That resistance is different in different reference frames with respect to which the particle moves with different velocities. Therefore the particle mass should also differ in different frames.

It should be stressed that the resistance (and therefore the increased resistance and energy) arises *in* the particle (more precisely, in the particle's worldtube); it does not come from the geometric properties of spacetime. It is spacetime that determines the shape of a geodesic worldline (and the shape of a geodesic worldtube in the case of a spatially extended particle), but it is the particle that *resists* when prevented from "following" a geodesic path, i.e., when the particle's worldtube is *deformed*.

We have proof that the resistance does not originate in the geometry of spacetime – a particle whose worldtube is deformed due to its deviation from its geodesic shape offers the *same* resistance in *both* flat and curved spacetime as the equivalence of inertial and passive gravitational masses shows (for more details see [39, Chap. 9]).

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