Quantum clocks in Minkowski space

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Time enters quantum theory through its appearance as an external classical parameter in the Schrödinger equation and is not treated on the same footing as other dynamical variables like position and momentum. In general relativity, time is defined operationally as what is indicated by a clock and the geometry of spacetime, as encoded in the metric tensor, determines the relationship between different clocks. Reconciling these two different notions of time in a quantum theory of gravity leads to the problem of time, one aspect of which is the disappearance of time in the Wheeler-DeWitt equation.

Motivated by this problem, the conditional probability interpretation (CPI) of time posits that the time evolution of a system of interest emerges from entanglement shared between the system and a clock, the joint state of which does not evolve with respect to a background notion of time and satisfies a Wheeler-DeWitt equation. The CPI can be constructed via the Dirac quantization of Rovelli's timeless relativistic formulation of classical mechanics and suggests a new quantum picture of spacetime depicted in the figure below.

After reviewing the CPI, I will present a generalization[1] in which the clock and system interact; we should expect such an interaction when the gravitational interaction between the clock and system is taken into account. I will demonstrate how such clock-system interactions result in a time-nonlocal modification to the Schrödinger equation, illustrating this modification with specific examples. Furthermore, by considering particles in Minkowski space with internal degrees of freedom that function as clocks, I will demonstrate how time dilation becomes probabilistic within the CPI framework and recover on average the special relativistic result[2].

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[1] A. R. H. Smith and M. Ahmadi, *Quantizing time: Interacting clocks and systems*, arXiv:1712.00081

[2] A. R. H. Smith and M. Ahmadi, *Quantum clocks in Minkowski space*, Forthcoming (2019)

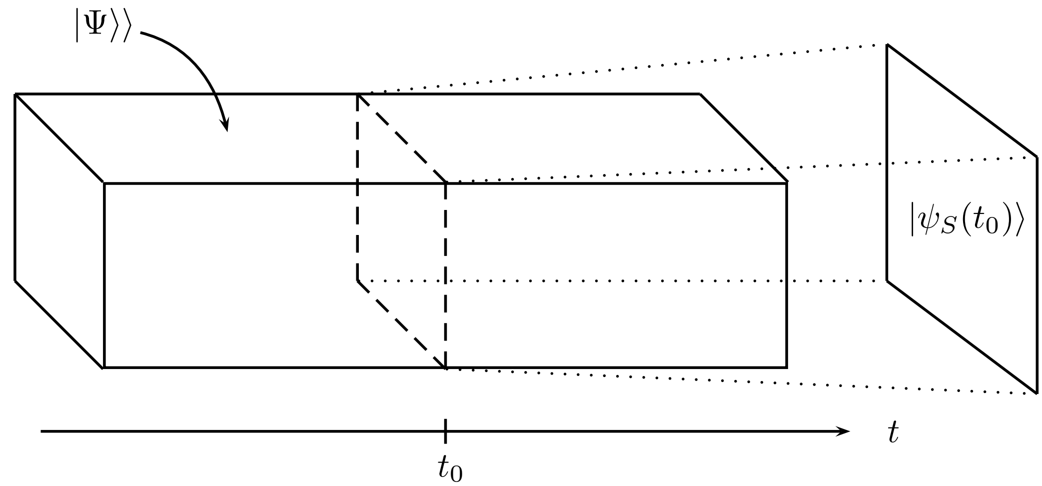

FIG. 1. The rectangular prism is a pictorial representation of the static state $|\psi\rangle\rangle = \int dt\ |t\rangle\,|\psi_S(t)\rangle$, which describes both a clock and system of interest. The horizontal axis represents the Hilbert space associated with the clock $\mathcal{H}_C$ and the directions orthogonal to the horizontal axis represent the Hilbert space of the system state $\mathcal{H}_S$. The system state $|\psi_S(t_0)\rangle$ at the time t_0 is obtained by conditioning $|\psi\rangle\rangle$ on the clock being in the state $|t_0\rangle$ and pictorially represented by a slice of the rectangular prism. This depiction of dynamics offered by the CPI might be thought of as a quantum block universe. Adapted from Giovannetti *et al.*, Phys. Rev. D 92, 045033 (2015).