Conformal invariance: the missing symmetry in general relativity theory?

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We investigate the conformal invariant Lagrangian of the self-gravitating U(1) scalar-gauge field on the time-dependent Bondi-Marder axially symmetric spacetime. By considering the conformal symmetry as exact at the level of the Lagrangian and broken in the vacuum, a consistent model is found with an exact solution of the vacuum Bondi-Marder spacetime, written as $g_{\mu\nu} = \omega^2 \bar{g}_{\mu\nu}$, where ω is the conformal factor and $\bar{g}_{\mu\nu}$ the 'un-physical' spacetime. Curvature could then be generated from Ricci-flat $\bar{g}_{\mu\nu}$ by suitable dilaton fields and additional gauge freedom. If we try to match this vacuum solution onto the interior vortex solution of the coupled Einstein-scalar-gauge field, we need, besides the matching conditions, constraint equations in order to obtain a topological regular description of the small-scale behaviour of the model. Probably, one needs the five-dimensional warped counterpart model, where the warp factor determines the large-scale behavior of the model. This warp factor is determined by the Einstein field equations for the five-dimensional warped space, where only gravity can propagate into the bulk. The warped five-dimensional model can be reformulated by considering the warp factor as a dilaton field conformally coupled to gravity and embedded in a smooth $M_4 \otimes R$ manifold. It is conjectured that the four-dimensional conformal factor is related to the dilaton field of the five-dimensional counterpart model. The dilaton field (alias warp factor), has a dual meaning. At very early times, when $\omega \to 0$, it describes the small-distance limit, while at later times it is a warp (or scale) factor that determines the dynamical evolution of the universe. However, as expected, the conformal invariance is broken (trace-anomaly) by the appearance of a mass term and a quadratic term in the energy-momentum tensor of the scalar-gauge field, arising from the extrinsic curvature terms of the projected Einstein tensor. These terms can be interpreted as a constraint in order to maintain conformal invariance and the tracelessness of the energy-momentum tensor could then be maintained by a contribution from the bulk. By considering the dilaton field and Higgs field on equal footing on small scales, there will be no singular behavior, when $\omega \to 0$ and one can deduce constraints to maintain regularity of the action. We also present a numerical solution of the model and calculate the (time-dependent) trace-anomaly. The solution depends on the mass ratio of the scalar and gauge fields, the parameters of the model and the vortex charge n.

INTRODUCTION

There is a urgent necessity for a description of gravity on small scales. Modifications of standard general relativity (GR) seems to be necessary in order to overcome the serious problems which one encounters when one decreases the scale closer to the Planck scale, where quantum effects come into play. This quantum approach has not yet been reconciled with the curved spacetime of GR. An elegant way to modify GR was given by Randall and Sundrum (RS), i.e., the warped five-dimensional spacetime[1–3]. The model can be regarded as the low-energy limit of general higher-dimensional theories which more fully address the particle interactions. Gravity can propagate freely into the 5D bulk, whereas the standard model fields are constrained to the 4D hypersurface. The hierarchy problem in these models would be solved and the cosmological constant and dark matter can be emergent[4, 5]. The characteristic warp factor in the RS-model can also be considered as a dilaton field in conformal invariant (CI) models[6–8].

CI in GR can be a promising formalism for disclosing the small-distance structure of GR. It can be considered as an exact local CI, spontaneously broken as in the case of the BEH mechanism. It is a controversial alternative method to describe canonical quantum gravity, because one is saddled with serious anomalies[9–11]. The key problem is perhaps the handling of asymptotic flatness of isolated systems in GR, specially when they radiate and the generation of the metric $g_{\mu\nu}$ from at least Ricci-flat spacetimes. Close to the Planck scale one should like to have Minkowski spacetime and somehow curvature must emerge. Curved spacetime will inevitable enter the field equations on small scales. The first task is then to construct a Lagrangian, where spacetime and the fields defined on it, are topological regular. This can be done by considering the scale factor(or warp factor in higher-dimensional models) as a dilaton field besides the conformally coupled scalar field. The same procedure can be applied to the 4D spacetime and one can try to generate from (Ricci)-flat spacetimes physical acceptable spacetimes in the non-vacuum case. It is known since the 70s[12], that quantum field theory combined with Einstein's gravity runs into serious problems. The Einstein-Hilbert(EH) action is non-renormalizable and it gives rise to intractable divergences at loop levels. On very small scales, due to quantum corrections to GR, one must modify Einstein's gravity by adding higher order terms in the Lagrangian like $R^2, R_{\mu\nu}R^{\mu\nu}$ or $R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau}$ (or combinations of them). However, serious difficulties arise in these higher-derivative models, for example, the occurrence of massive ghosts which cause unitary problems. A next step is then to disentangle the functional integral over the dilaton field from the ones over the metric fields and matter fields. Moreover, it is desirable that all beta-functions of the matter lagrangian, in combination with the dilaton field, disappear in order to fix all the coupling constants of the model. Further, conformal invariance of the action with matter fields implies that the trace of the energy-momentum tensor is zero. A theory based on a classical "bare" action which is conformally invariant, will lose it in quantum theory as a result of renormalization and the energy-momentum tensor acquires a non-vanishing trace (trace anomaly). We consider here the breaking of conformal invariance in conventional Einstein theory and will not enter into details of these quantum-gravity problems. It is conjectures that conformal symmetry is exact at the level of the Lagrangian and only broken in the vacuum, just as the BEH mechanism in standard model of particle physics. This approach can even be an alternative to supersymmetry and the dark energy problem. Because our axially symmetric model can easily transformed to spherical symmetry, it is clear that our conformal invariant study of the self-gravitating coupled scalar-gauge field on an axially symmetric spacetime make sense in studying the small scale properties. We describe here an example of CI on an axially symmetric Bondi-Marder spacetime with a U(1) gauged scalar field in the interior.

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