Conformal Gravity

The missing symmetry in GR?

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Hermann Minkowski

$$d\tau = \sqrt{dt^2 - dx^2 - dy^2 - dz^2}$$

Slagter, Foud Phys **2016**; Phys.Dark Universe **2019**
Slagter, arXiv: gr-qc/190206088V4 subm. to Ann. of Phys.**2019**
[Further reading: **‘t Hooft**, **arXiv:2009, 2011,2015**]

Motivations for Conformal Invariant Gravity

1. **Mainly quantum-theoretical: opportunity for a renormalizable theory with preservation of causality and locality [alternative for stringtheory?]**

note: *"formulating GR as a gauge group was not fruitful", so "add" CI to gauge*

2. **Formalism for disclosing the small-distance structure in GR**

Note *"there seems to be no limit on the smallness of fundamental units in one particular domain of physics, while in others there are very large scales and time scale"*

consider: **local exact CI, spontaneously broken just as the Higgs mechanism**

3. **CI can be used for** *"black hole complementarity"* **and information paradox [related to holography [**'t Hooft** 1993, 2009]**

4. **Alternative to dark energy/matter issue** [**Mannheim**, 2017];
Construct traceless $T_{\mu\nu}$ [needed for CI: particles massless] and use spontaneous symmetry breaking!

5. **Explore issues such as "trans-Planckian" modes in Hawking radiation calculation and the nature of "entanglement entropy"**

Example: warped 5D model: dilaton from 5D Einstein eq [**Slagter**, 2016]

Some results of Conformal Invariance

- CI in GR should be a ***spontaneously broken exact symmetry***, just as the Higgs mechanism
- **One splits the metric:** $\boldsymbol{g_{\mu\nu}(x) = \omega(x)^2 \tilde{g}_{\mu\nu}(x)}$ $\boldsymbol{\tilde{g}_{\mu\nu}}$ **the "unphys. metric"**

treat ω and scalar fields on equal footing!

- CI is well define on Minkowski: null-cone structure is preserved.
- If $\tilde{g}_{\mu\nu}$ *is (Ricci?) flat*: ω is unique (QFT is done on flat background!)
- If $\tilde{g}_{\mu\nu}$ *is non-flat*: additional gauge freedom: $\boldsymbol{\tilde{g} \to \Omega^2 \tilde{g}, \ \omega \to \frac{1}{\Omega}\omega,}$ $\Phi \to \frac{1}{\Omega}\Phi, \ldots\ldots\ldots$

  [no further dependency on Ω, ω]

  SO: can we generate $\boldsymbol{\tilde{g}_{\mu\nu} = \Omega^2 \eta_{\mu\nu}}$? ***I will present 2 examples*** (see next)

- **conjecture**: avoiding *anomalies* we generate constraints which will determine the physical constants such as the *cosmological constant*
- Consider conformal component of metric as a ***dilaton*** (ω)with only renormalizable interactions.
- **Small distance behavior** (ω⟶0) regular behavior by imposing *constraints* on model
- **Spontaneously breaking:** *fixes all parameters* (mass, cosm const,…) **['t Hooft, 2015]**

Some results of Conformal Invariance

► Dilaton field ω need to be ***shifted to complex*** contour (Wick rotation) to ensure that ω has the same *unitary* and positivity properties as the scalar field.
[**for our 5D model**: ω has ***complex solutions***!]

► In canonical gravity**:** ***quantum amplitudes*** are obtained by integration of the action over all components of $\boldsymbol{g_{\mu\nu}}$.

Now: first over $\boldsymbol{\omega}$; and then over $\tilde{g}_{\mu\nu}$; **then:** constraints on $\tilde{g}_{\mu\nu}$ and matter fields

$$\int d^5W \int d^4\omega \int d\tilde{g}_{\mu\nu} \ldots\ldots e^{iS}$$

[$\tilde{g}_{\mu\nu}$still inv. under local conv. trans.]

S gauge fixing constraints.

►Vacuum state would have normally **R=0**; **now:** $\boldsymbol{R \to \frac{R}{\Omega^2} - \frac{6}{\Omega^3} \nabla^\mu \nabla_\mu \Omega}$

so the vacuum breaks local CI spontaneously

Nature is not scale invariant, so the vacuum transforms into another unknown state.

►**Conjecture: conformal anomalies must be demanded to cancel out**

→ **all renormalization group β-coeff must vanish**

→ **constraints to adjust all physical constants!**

►**Ultimate goal: all parameters of the model computable (including masses and Λ)**

Severe problems of GR

Major problems: 1. Hiarchy-problem (why is gravity so weak?)

2. What is dark-energy (needed for accelerated universe) Λ needed??

3. Then: huge discrepancy between $\rho_\Lambda \sim 10^{-120}$ and $\rho_{vac.} \sim 10^{-3}$
+ incredibly fine-tuned: $\Omega_\Lambda \sim \Omega_{Mat}$

4. What happens at the Planck length? TOE possible?

5. The black hole war: Hawking--'t Hooft
Desperately needed: quantum-gravity model

6. Do we need higher-dimensional worlds?
[are we a "hologram"]

NOW: 7. How do we make gravity conformal (scale-) invariant?

- ■ alternative for disclosing the small-distance structure of GR
- ■ No dark energy (matter?) necessary [Mannheim, 't Hooft]
- ■ CI a local symm, spontaneously broken in the EH-action[as the BEH] ?

Some history of QFT

Calculations in QFT:

- *In perturbation theory the effect of interactions is expressed in a powerserie of the coupling constant (<<1 !)*
- *Regularization scheme necessary in order to deal with divergent integrals over internal 4- momenta.*
- *Introduce cut-off energy/mass scale Λ and stop integration there.*

  [however, invisible in physical constants and partcle data tables]

  So renormalization comes in
- *Covariant theory of gravitation cannot be renormalized [in powercounting sense]*

  Non-renormalizable interactions is suppressed at low energy, but grows with energy. At energies much smaller than this "catastrophe-scale", we have an effective field theory.

Standard model is too an effective field theory.

- **In curved background: geometry of spacetime remaims in first instance non-dynamical!**

  However: in GRT it is.

String theory solution?

- **Nambu-Goto action (Polyakov)** $A = -T\int d^2\sigma\sqrt{-g}\,g^{\alpha\beta}h * \eta_{\alpha\beta}$

Some history of QFT

New gauge symmetry: $g_{\alpha\beta} \to \Omega(\sigma)^2 g_{\alpha\beta}$ [Ω smooth function on the worldsheet]

After quantization: $\langle T^{\alpha}_{\alpha} \rangle$ depends on Ω, unless a crucial number in 2d-CFT (central charge) is zero! [in conformal gravity $T^{\alpha}_{\alpha} = 0$]

The Fadeev-Popov ghost field (needed for quantisation) contribute a central charge of -26, which can be canceled by 26-dimensional background.

Can we do better? **New conformal field theory**

Suppose: QFT is correct and GRT holds at least to the Planck scale

- **Advantages of CI:**

A. ***At high energy, the rest mass of partcles have negligible effects***
So no explicit mass scale. ***CI would solve this***

B. CI field theory ***renormalizable*** [coupling constants are dimensionless]

C. CI In curved spacetime: would solve the ***black hole complementarity*** through conformal transformations between infalling and stationary observers.

D. Could be ***singular-free***

E. Success in CFT/ADS correspondence

F. In standart model, symmetry methods also successful.

G. CI put constraints on GRT . Very welcome!

Related Issues

► If spacetime is fundamental discrete: then continuum symmetries (such as L.I.) are imperilled. To make it compatible: the price is locality. [**Dowker**, 2012; **'t Hooft**, 2016]

Can non-locality be tamed far enough to allow known local physics to emerge at large distances?

► The Causal Set approach to quantum gravity: atomic spacetime in which the fundamental degrees of freedom are discrete order relations. [**'tHooft, Myrheim, Bombelli, Lee, Myer and Sorkin**]

► The causal set approach claims that certain aspects of General Relativity and quantum theory will have direct counterparts in quantum gravity:

1. the spacetime causal order from General Relativity,
2. the path integral from quantum theory.

Then: Is it possible to obtain our familiar physical laws described by PDE's from discrete diff operators on causal sets? For example, discrete operators that approximate the scalar D'Alembertian in any spacetime dimension? Seems to be yes!

►ω is fixed when we specify our global spacetime and coordinate system, which is associated with the vacuum state.

[remember $\boldsymbol{R \rightarrow \frac{R}{\Omega^2} - \frac{6}{\Omega^3} \nabla^\mu \nabla_\mu \Omega}$ **]** If we not specify this state, then no specified ω.

<u>**'t Hooft**</u>: *" In quantum field theory we work on a flat background. Then ω is unique On non-flat background: sizes and time stretches and become ambiguous"*

Related Issues

- **Asymptopia:** **How to handle:** **"far from an isolated source?"**
  we have only locally: $\boxed{\nabla_\alpha T^{\alpha\beta} = 0}$
  is there a Killing-vector k_μ: then $\boxed{\nabla_\alpha J^\alpha = \nabla_\alpha\left(T^{\alpha\beta}k_\beta\right) = 0}$
  then integral conservation law.
  gravitational energy and mass?
- Isotropic scaling trick: $g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu} = \omega^2 g_{\mu\nu}$ with $\omega \rightarrow 0$ far from the source.

[**note:** we shall see that Einstein equations yield: $G_{\mu\nu} = \frac{1}{\omega^2}(\dots)$, so small distance limit will cause problem, unless we add scalar field comparable with "dilaton "ω : $G_{\mu\nu} = \frac{1}{\omega^2+\Phi^2}(\dots)$]

Example: Minkowski: $\boldsymbol{ds^2 = -dvdu + \frac{1}{4}(v-u)^2\left[d\theta^2 + \sin^2\theta d\varphi^2\right]}$

one needs information about behavior of fields at $v \rightarrow \infty$

then: $\boldsymbol{ds^2 = \frac{1}{V^2}\left[dudV + \frac{1}{4}(1-uV)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]}$ and infinity : $V \rightarrow 0$

so singular!

then: $\boldsymbol{g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu} = \omega^2\eta_{\mu\nu} = V^2\eta_{\mu\nu}}$ *: smooth metric extended to V=0 and one can handle tensor analysis at infinity.*

Even better: $\boldsymbol{\hat{g}_{\mu\nu} = \frac{4}{(1+v^2)(1+u^2)}\eta_{\mu\nu}}$ with $T, R = \tan^{-1}v \pm \tan^{-1}u$

$$\boldsymbol{ds^2 = -dT^2 + dR^2 + \sin^2 R\left(d\theta^2 + \sin^2\theta d\varphi^2\right)}$$

Static Einstein universe $S^3 \otimes \mathcal{R}$: conformal map $(\mathcal{R}^4, \eta_{\mu\nu}) \rightarrow (S^3 \otimes \mathcal{R}, \hat{g}_{\mu\nu})$

Connection with 5D Warped Spacetime

Consider on a 5D warped spacetime [NOT yet CI] [Slagter,2016]

$$ds^2 = \mathcal{W}(t,r,y)^2\left[e^{2(\gamma(t,r)-\psi(t,r))}\left(-dt^2+dr^2\right)+e^{2\psi(t,r)}dz^2+r^2e^{-2\psi(t,r)}d\varphi^2\right]+\Gamma dy^2$$

U(1) scalar-gauge field on the brane + empty bulk. Gravity can propagate into the bulk.

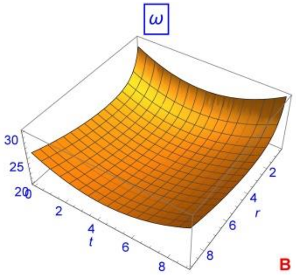
5D:

$${}^5G_{\mu\nu} = -\Lambda_5\,{}^5g_{\mu\nu} + \kappa_5^2\delta(y)\left[-{}^4g_{\mu\nu}\Lambda_4 + {}^4T_{\mu\nu}\right]$$

On the brane:

$${}^4G_{\mu\nu} = -\Lambda_{eff}\,{}^4g_{\mu\nu} + \kappa_4^2\,{}^4T_{\mu\nu} + \kappa_5^4\,S_{\mu\nu} - \mathcal{E}_{\mu\nu}$$

From 5D:



$$\mathcal{W} = \frac{e^{\sqrt{-\frac{1}{6}\Lambda_5}(y-y_0)}}{\alpha\sqrt{r}}\sqrt{(d_1e^{\alpha t}-d_2e^{-\alpha t})(d_3e^{\alpha r}-d_4e^{-\alpha r})}$$

$$\Phi = \eta X(t,r)e^{in\varphi}, \qquad A_\mu = \frac{1}{\epsilon}[P(t,r)-n]\nabla_\mu\varphi$$

Scalar-gauge field eq.:

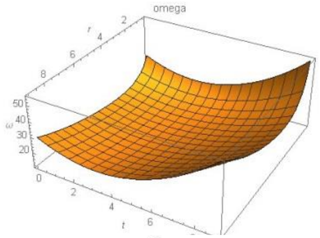
$$D^\mu D_\mu\Phi = 2\frac{\partial V}{\partial\Phi^*} \qquad {}^4\nabla^\mu F_{\mu\nu} = \frac{1}{2}i\epsilon[\Phi(D_\nu\Phi)^*-\Phi^*D_\nu\Phi]$$

One could say that the "**information about the extra dimension**" translates itself as a curvature effect on spacetime of one fewer dimension!!

Warped 5D spacetime conformally revisited

We rewrite our metric

$$ds^2 = \omega(t,r)^2 W(y)^2 \tilde{g}_{\mu\nu} + n_\mu n_\nu \Gamma(y)^2$$

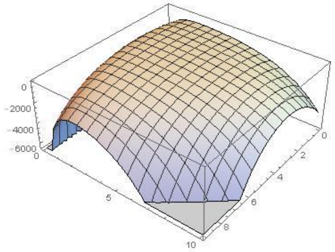


← real solution.

↓ ↓

dilaton **“unphysical metric”** [Bondi-Marden.]

$$(\partial_{tt}-\partial_{rr}-\frac{2}{r}\partial_r)\omega+\frac{\partial_r\omega^2-\partial_t\omega^2}{\omega}=0$$



← solution: $\omega^2 < 0$ needed : integration over complex contour[‘tHooft..]
and ω has same unitary and positivity prop as Φ

write the action **conformal invariant** **[i.e. : $\tilde{g}_{\mu\nu} \to \Omega^2 \tilde{g}_{\mu\nu}$ $\quad \bar{\omega} \to \frac{1}{\Omega}\bar{\omega}$ $\quad \tilde{\Phi} \to \frac{1}{\Omega}\tilde{\Phi}$]**

$$A = \int d^4x \sqrt{-\tilde{g}}\left[-\frac{1}{12}(\Phi\Phi^* + \bar{\omega}^2)\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}(\tilde{\partial}_\mu\bar{\omega}\tilde{\partial}_\nu\bar{\omega} + D_\mu\tilde{\Phi}D_\nu\tilde{\Phi}^*)\right] - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}$$

$$- V(\tilde{\Phi},\bar{\omega}) - \frac{1}{36}\kappa_4^2\Lambda\bar{\omega}^4 \Big]$$

$$\omega^2 = -\frac{1}{6}\kappa_4^2\bar{\omega}^2$$

$$V(\tilde{\Phi},\bar{\omega}) = \frac{1}{8}\beta\eta^2\kappa_4^2\tilde{\Phi}\tilde{\Phi}^*\bar{\omega}^2 + \lambda\tilde{\Phi}^4$$

Note:
- * CI broken by mass term via $V(\tilde{\Phi},\bar{\omega})$
- * we take $\Lambda=0$
- * Newton’s const hidden in $V(\tilde{\Phi},\bar{\omega})$, so **re-appears** when CI is **broken**

Warped 5D spacetime conformally revisited

Field equations rewritten[**Slagter**,2019]

$$\tilde{G}_{\mu\nu} = \frac{1}{\left(\bar{\omega}^2 + \tilde{\Phi}\tilde{\Phi}^*\right)}\left[\tilde{T}_{\mu\nu}^{(\bar{\omega})} + \tilde{T}_{\mu\nu}^{(\tilde{\Phi},c)} + \tilde{T}_{\mu\nu}^{(A)} + \frac{1}{6}\tilde{g}_{\mu\nu}\Lambda_{eff}\kappa_4^2\bar{\omega}^4 + \kappa_5^4 S_{\mu\nu} + \tilde{g}_{\mu\nu}V(\tilde{\Phi},\bar{\omega})\right] - \mathcal{E}_{\mu\nu}$$

$$\tilde{\nabla}^\alpha\tilde{\partial}_\alpha\bar{\omega} - \frac{1}{6}\bar{\omega}\tilde{R} - \frac{\partial V}{\partial\bar{\omega}} - \frac{1}{9}\Lambda_4\kappa_4^2\bar{\omega}^3 = 0$$

$$D^\alpha D_\alpha\tilde{\Phi} - \frac{1}{6}\tilde{\Phi}\tilde{R} - \frac{\partial V}{\partial\tilde{\Phi}^*} = 0$$

$$\tilde{\nabla}^\nu F_{\mu\nu} = \frac{i}{2}e\left(\tilde{\Phi}\left(D_\mu\tilde{\Phi}\right)^* - \tilde{\Phi}^* D_\mu\Phi\right)$$

Calculate **Trace**: rest term as expected:

$$\frac{1}{\bar{\omega}^2+X^2}[16\kappa_4^2\beta\eta^2X^2\bar{\omega}^2 \;-\; \kappa_5^4\left(\frac{\partial_r P^2-\partial_t P^2}{r^2e^2}\right)^2 e^{8\tilde{\psi}-4\tilde{\gamma}}]$$

Bianchi: $\nabla^\mu\,\mathcal{E}_{\mu\nu}{=}\kappa_5^4\,\nabla^\mu\,S_{\mu\nu}$ so (3+1) spacetime variation in matter-radiation on brane can source KK modes

$$\tilde{T}_{\mu\nu}^{(\bar{\omega})} = \tilde{\nabla}_\mu\partial_\nu\bar{\omega}^2 - \tilde{g}_{\mu\nu}\tilde{\nabla}_\alpha\partial^\alpha\bar{\omega}^2 - 6\partial_\mu\bar{\omega}\partial_\nu\bar{\omega} + 3\tilde{g}_{\mu\nu}\partial_\alpha\bar{\omega}\partial^\alpha\bar{\omega}$$

$$\tilde{T}_{\mu\nu}^{(\tilde{\Phi},c)} = \tilde{\nabla}_\mu\partial_\nu\tilde{\Phi}\tilde{\Phi}^* - \tilde{g}_{\mu\nu}\tilde{\nabla}_\alpha\partial^\alpha\tilde{\Phi}\tilde{\Phi}^* \;- 3\left(\mathcal{D}_\mu\tilde{\Phi}(D_\nu\tilde{\Phi})^*{+}(D_\mu\tilde{\Phi})^*D_\nu\tilde{\Phi} + 3\tilde{g}_{\mu\nu}D_\alpha\tilde{\Phi}(D^\alpha\tilde{\Phi})^*\right)$$

$$\tilde{T}_{\mu\nu}^{(A)} = F_{\mu\alpha}F_\nu^\alpha - \frac{1}{4}\tilde{g}_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$$

New: Some applications

We will consider now two examples of the "**un-physical**" metric $\tilde{g}_{\mu\nu}$

A. **Bondi-Marder spacetime** [suitable for our scalar-gauge model]

I. With the contribution from projected Weyl tensor [**Slagter** ,ArXiv:gr-qc/171108193]
II. Without [**Slagter**, Phys Dark Universe,2019]

B. **Spinning Cosmic String** [Bonner: "*urgent need convincing phys interp of CTC's ..*"]

Stationary axially symmetric solutions: Kerr solution. CTC's hidden behind the horizon
Where are the others?

Weyl, Parapetrou, van Stockum, ... All are physically unacceptable: not the correct asymptotic behavior
CTC's are possible
matching problems at the boundary

However: cosmic string solution in GR : could be physically acceptable .

Now: spinning cosmic strings: Some additional fields are necessary to compensate the energy failure close to the core.

THEN: How do we solve the CTC problem and matching problem??

By Conformal invariant model?

Bondi-Marder spacetime as "unphysical" metric

Remember: Bondi-Marder spacetime [needed because $T_{tt} + T_{rr} \neq 0$ for CS]

$$ds^2 = e^{-2\psi}[e^{2\gamma}(dr^2 - dt^2) + r^2 d\varphi^2] + e^{2\psi+2\mu}dz^2$$
$$= \hat{\omega}^2[-dt^2 + dr^2 + e^{2\tau}dz^2 + r^2 e^{-2\gamma}d\varphi^2]$$

So $$\tilde{g}_{\mu\nu} = \hat{\omega}^2 \bar{g}_{\mu\nu}$$

$\uparrow$ $\uparrow$ Ricci-flat

un-physical metric from 5D

$\hat{\omega}$ is a conformal factor. We consider first the exterior vacuum situation:

Einstein equation: $$\hat{\omega}^2 \bar{G}_{\mu\nu} = T_{\mu\nu}^{(\hat{\omega})}$$

$\hat{\omega}$ - equation: $$\bar{\nabla}^\mu \partial_\mu \hat{\omega} - \frac{1}{6}\hat{\omega}\bar{R} = 0$$

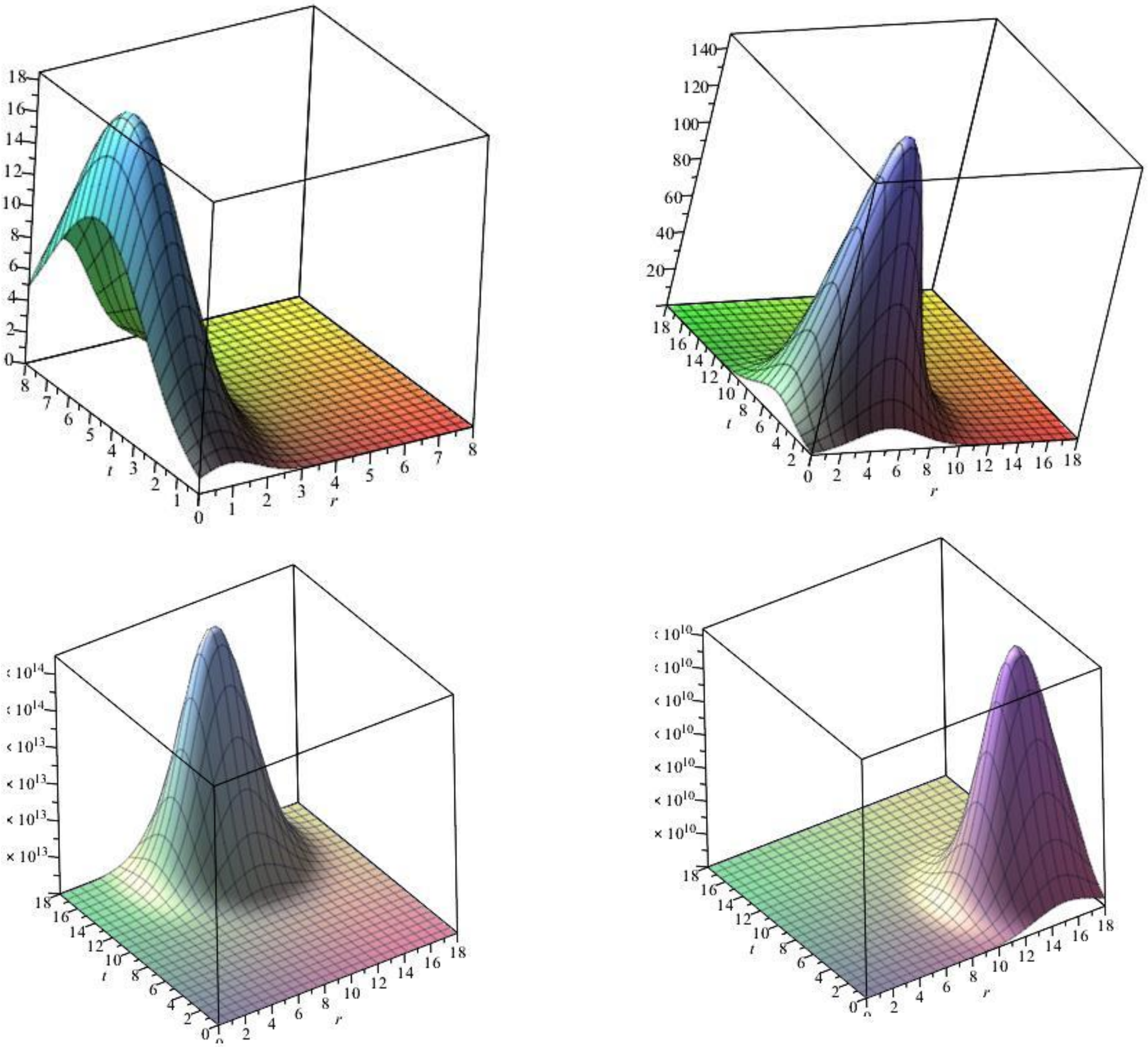
Check: $$Tr\left[\bar{G}_{\mu\nu} - \frac{1}{\hat{\omega}^2} T_{\mu\nu}^{(\hat{\omega})}\right] = 0$$

One can solve equation for $\hat{\omega}$:

$$\hat{\omega} = \mathcal{B}e^{\frac{1}{2}\varsigma_1(r^2+t^2) - \frac{1}{2}\upsilon r^2 + \varsigma_2 t + r}$$

4 constants . Generation of curvature from Ricci flat spacetimes. [Slagter, Phys. Dark Univ.,2019]

Numerical solution ω



Quantum amplitudes are obtained by

$$\int D\omega(x)\ \ldots..$$

No problem here.

Spinning U(1) gauged cosmic strings

Let us consider now the 4D stationary axially symmetric spacetime with **rotation:**
[for the moment no t-dependency]

$$ds^2 = -e^{-2f(r)}(dt - J(r)d\varphi)^2 + e^{2f(r)}\left[l(r)^2 d\varphi^2 + e^{2\gamma(r)}(dr^2 + dz^2)\right]$$

rewritten as

$$\boldsymbol{ds^2 = \omega(r)^2\left[-(dt - J(r)d\varphi)^2 + b(r)^2 d\varphi^2 + e^{2\mu(r)}(dr^2 + dz^2)\right]}$$

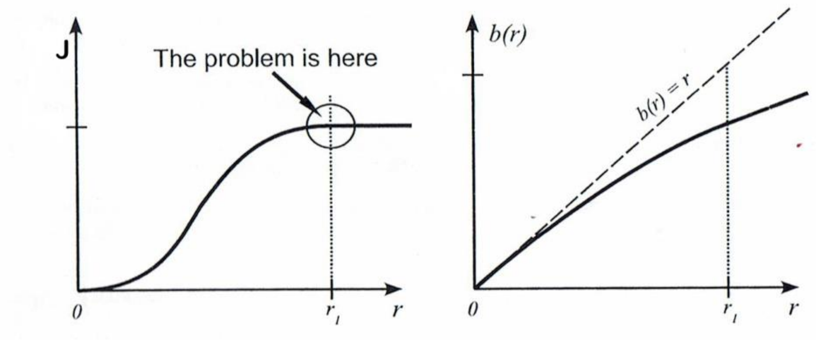
Some results:
1. obtainable from Weyl form by: $t \to iz, \quad z \to it, \quad J \to iJ$
2. interesting relation with (2+1) dim gravity [cosmon's; **'tHooft ,**2000]
3. Gott-spacetime: no CTC's [parallel and opposite moving pair]
4. for constant J: ► conical exterior spacetime [**angle-deficit**]
   ►if one transform: $t \to t - J\varphi$: results in local Minkowski
   but then t jumps by $8\pi GJ$ **[helical time**]
   QM-solution? Quantized angular momentum→ also t !
5. What happed at the **boundary** r_c of the string?

r=0: J = 0 and b→r

$r = r_c$: J = constant and $b = B(r + r_c)$

Then:

problems at the boundary for J_r and **WEC** violated!!



Spinning U(1) gauged cosmic strings in CI gravity

No choice yet for $V(\omega,\Phi)$. From tracelessness and Bianchi:

$$\frac{2}{3}V=\tilde{\Phi}^{*}\frac{dV}{d\tilde{\Phi}^{*}}+\hat{\omega}\frac{dV}{d\hat{\omega}} \qquad \frac{1}{6}V'=\tilde{\Phi}^{*\prime}\frac{dV}{d\tilde{\Phi}^{*}}+\hat{\omega}'\frac{dV}{d\hat{\omega}}$$

For the **exterior** we obtain

$$J''=J'\left(\frac{b'}{b}-2\frac{\hat{\omega}}{\hat{\omega}}\right) \qquad b''=\frac{1}{b}J'^{2}-\frac{2}{\hat{\omega}}b'\hat{\omega}' \qquad \mu''=\frac{1}{2b^{2}}J'^{2}-\mu'\left(\frac{b'}{b}+2\frac{\hat{\omega}'}{\hat{\omega}}\right)$$

$\downarrow$ **"spin-mass rel"**

$$\hat{\omega}''=-\frac{3\hat{\omega}}{8b^{2}}J'^{2}+\frac{\hat{\omega}'^{2}}{2\hat{\omega}}+\frac{1}{2}\mu'\left(\frac{\hat{\omega}'b'}{b}+2\hat{\omega}'\right)$$

$$\boldsymbol{J(r)=\text{const.}\int\frac{b}{\hat{\omega}'^{2}}\,dr}$$

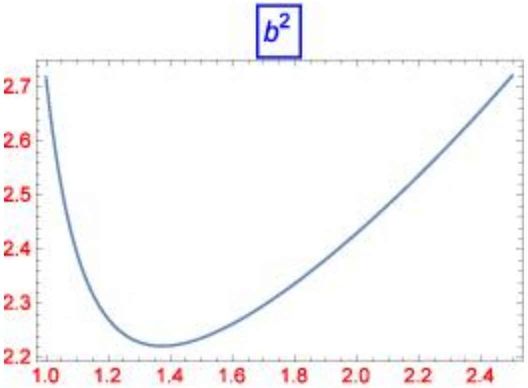
with **exact** solution:

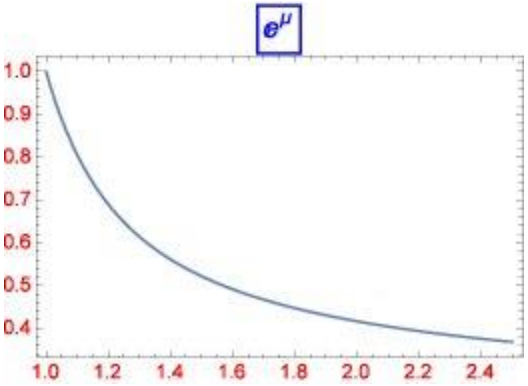
$$\boldsymbol{\mu(r)=c_1 r+c_2-\log\left(\sqrt{c_4 r+c_5}\right)} \qquad \boldsymbol{b(r)=\frac{c_3}{2c_4 r+2c_5}} \qquad \boldsymbol{\omega(r)=\sqrt{2c_4 r+2c_5}}$$

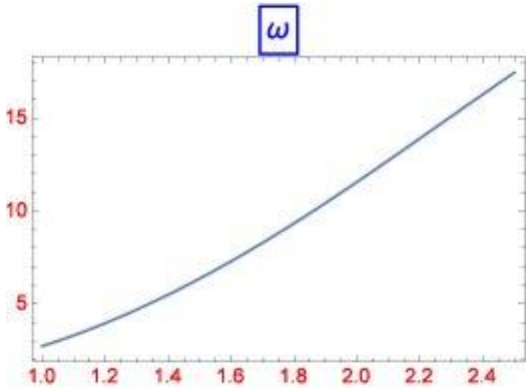
$$\boldsymbol{J(r)=c_6\pm\frac{c_3}{2c_4 r+2c_5}}$$

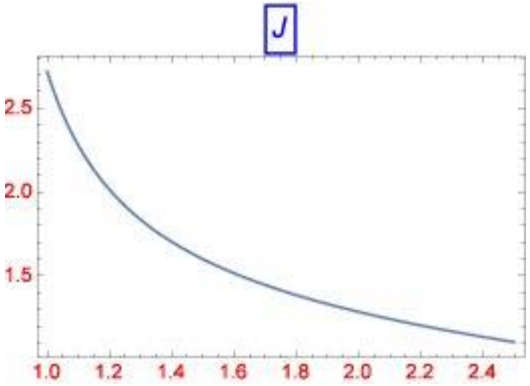
- ► J has correct asymptotic form!
- ► Ricci flat! [from the **inverse**: $\tilde{g}_{\mu\nu}=\frac{1}{\hat{\omega}^{2}}\,g_{\mu\nu}$ gen of non-flat from Ricci flat]
- ► CTC for $r=\frac{c_3-c_5c_6}{c_4c_6}$ which can be **pushed to ±∞**. [c_6 small)

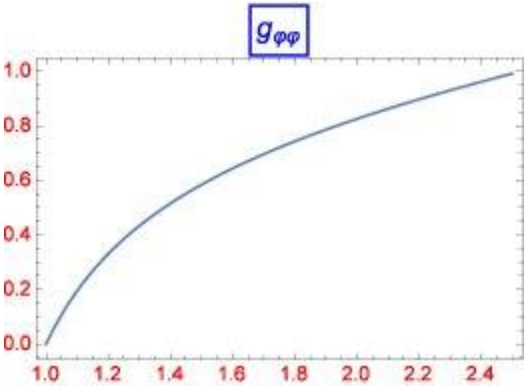
Numerical verification

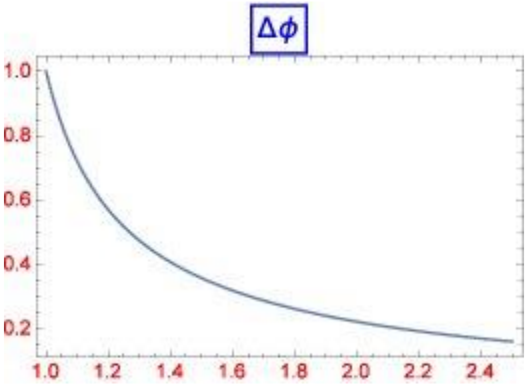

b^2


e^{μ}


ω


J


$g_{\varphi\varphi}$


$\Delta\phi$

The interior solution

For the gauge field we can take: $\boldsymbol{A_\mu = \left[P_0(r), 0, 0, \frac{1}{e}(P(r) - n)\right]}$

The field equation contain now terms like

$$J'' = J'\ \partial_r \left[log \left(\frac{b}{\eta^2 X^2 + \widehat{\omega}^2} \right) \right] - 2\frac{P_0'(eJP_0' + P')}{e(\eta^2 X^2 + \widehat{\omega}^2)} + \cdots$$

The **"spin-mass"** relation becomes in case of **global strings** (P=P_0 =0)

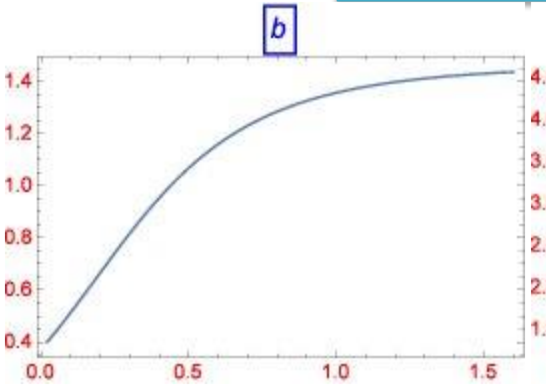
$$J = const \int \frac{b}{\eta^2 X^2 + \widehat{\omega}^2}\, dr$$

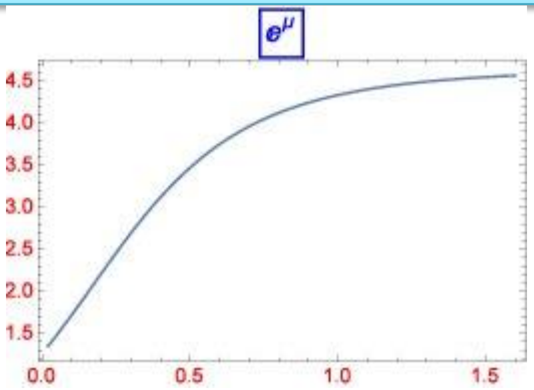
Energy momentum:

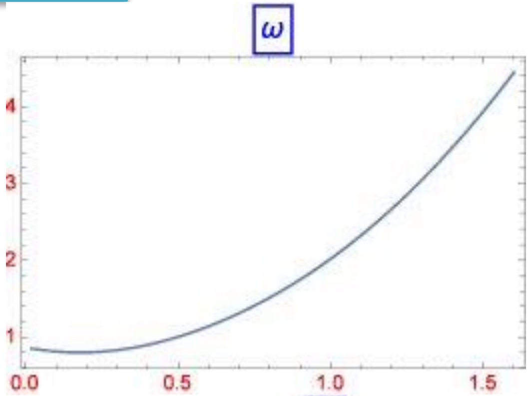
$$T_{tt} = -\frac{3}{4b^2} J'^2 + \frac{\mu' b'}{b} + (\mu' + \frac{b'}{b})\partial_r(\log(\eta^2 X^2 + \widehat{\omega}^2))$$

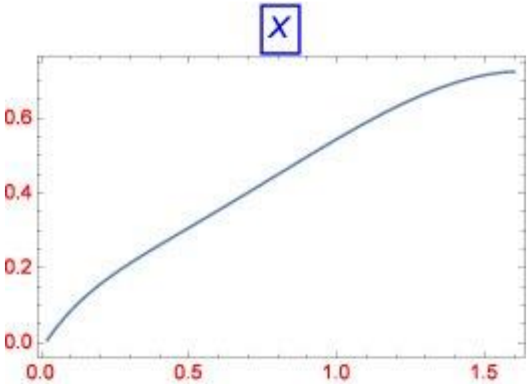
This can be made positive due to the additional matter!

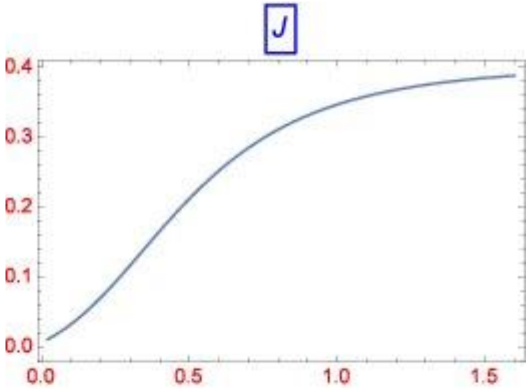
Numerical solution

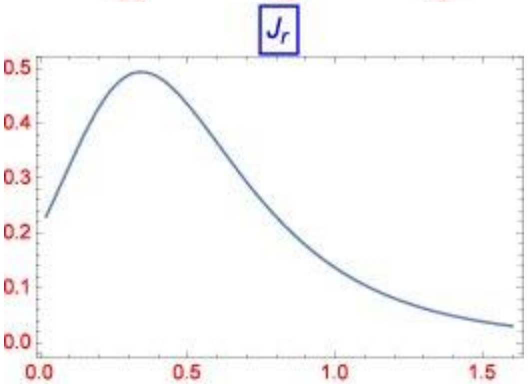


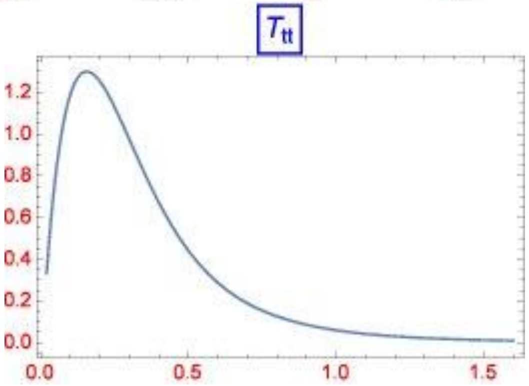


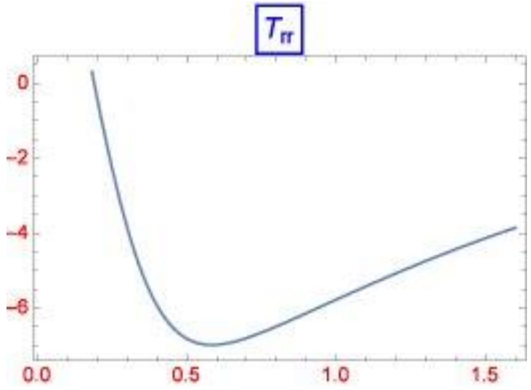


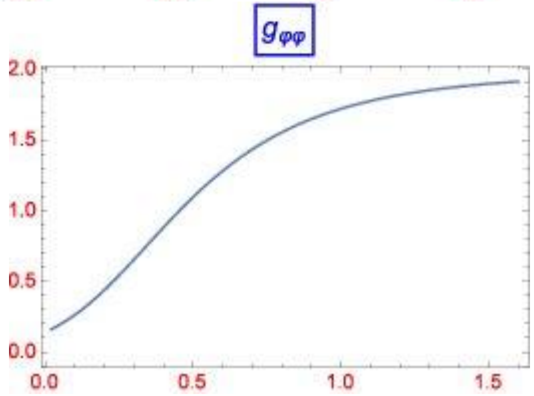












Local observer

Local orthonormal frame: $\widehat{\Theta}^t = dt - J d\varphi \quad \widehat{\Theta}^r = e^\mu dr \quad \widehat{\Theta}^z = e^\mu dz \quad \widehat{\Theta}^\varphi = b d\varphi$

Timelike 4-velocity: $U_{\hat{\nu}} = \frac{1}{\varepsilon}[1,0,\alpha,\beta]$

Local energy density measured by the observer moving at constant $r = r_c$

$$\varepsilon^2 G^{\hat{\mu}\hat{\nu}} U_{\hat{\mu}} U_{\hat{\nu}} = \frac{(\beta^2+\alpha^2)b' + \beta J'}{b} \partial_r [log(\eta^2 X^2 + \hat{\omega}^2)] + \frac{2\alpha^2 - \varepsilon^2}{4b^2} J'^2$$

Can be made ***positive*** for suitable physically acceptable behavior of b', J', X', ω'
and $\varepsilon^2 < 2\alpha^2$ (for sufficiently high velocity)

==

►► It seems that there are no obstructions for a physically acceptable solution
for a spinning cosmic string in conformal gravity.

Summary

A. ***Warpfactor*** W reinterpreted as ***dilaton*** **ω** from vacuum 5D Einstein equations of $M_4 \otimes R$

B. Warpfactor [exact solution] has **dual meaning** in ***CI GR*** model:

ω→ 0: dilaton describes the small distance limit

Now: **ω** is also scale factor, determines the dynamical evolution of universe.

C. By considering dilaton and scalar field on equal footing: ***no singular behavior as*** $\omega \rightarrow 0$

D. CI is broken (*trace-anomaly*) by mass terms in EH action.

However: in warped 5D model: contribution from ***quadratic terms in*** $T_{\mu\nu}$

SO: extra ***constraints*** in order to maintain ***tracelessness.***

E. Examples:

► On ***Bondi Marder*** ST (axially symmetric) ***curvature generation*** from Ricci-flat ST.

using ***additional gauge freedom:*** $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$; $\omega \rightarrow \frac{1}{\Omega}\omega$; $\Phi \rightarrow \frac{1}{\Omega}\Phi$

Necessary as a conformal gauge in order to make a renormalizable model.

► ***Spinning (global) cosmic strings:*** **asymptotic correct**

interior matches on exterior

no CTC's and WEC fulfilled

New indication that local CI make sense