PLANCK STARS
A LQG TRANSITION

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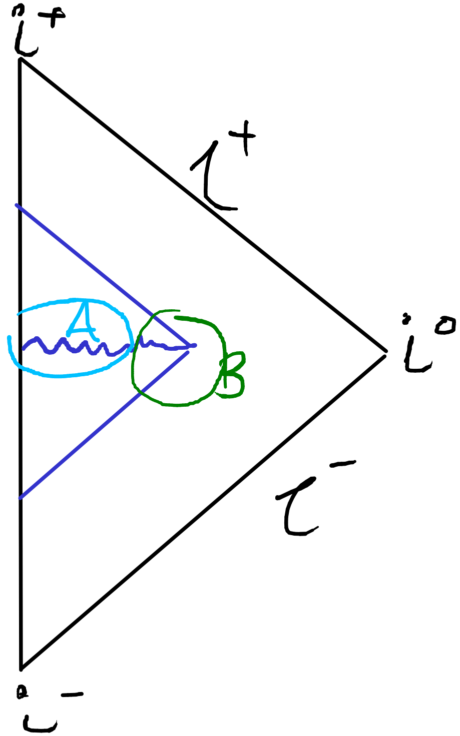

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LQG First 25 years, C. Rovelli
arxiv 1012.4707

Loop Quantum Gravity: The Beginning

Start with Einstein-Hilbert Action:

$$S(g) = \frac{1}{16\pi G} \int \sqrt{-g}\,(R - 2\Lambda)\, d^4x$$

$$\text{E-L eq's} \Rightarrow G_{\mu\nu} = 8\pi T_{\mu\nu}$$

First problem: can't couple a dirac equation to this! So introduce local frames, The Tetrads:

$$g_{\mu\nu}(x) = e^I_\mu(x)\, e^J_\nu(x)\, \eta_{IJ} \quad \text{Mink metric.}$$

Using these, one can convert the EH action into the Holst action:

$$S(e, \omega) = \int B(e) \wedge F(\omega)$$

Torsion

e – tetrads

ω – local lorentz gauge

* Covariant Loop Quantum Gravity, C. Rovelli, F. Vidotto, 2014

$$B = \left(*e \wedge e + \frac{1}{\gamma} e \wedge e\right)$$

(In general, if you only have $B(e)$ then this is called BF theory, but it is purely gauge)

γ = Barbero-Immirzi parameter.

To make sure this action still describes General Relativity, one must ensure that:

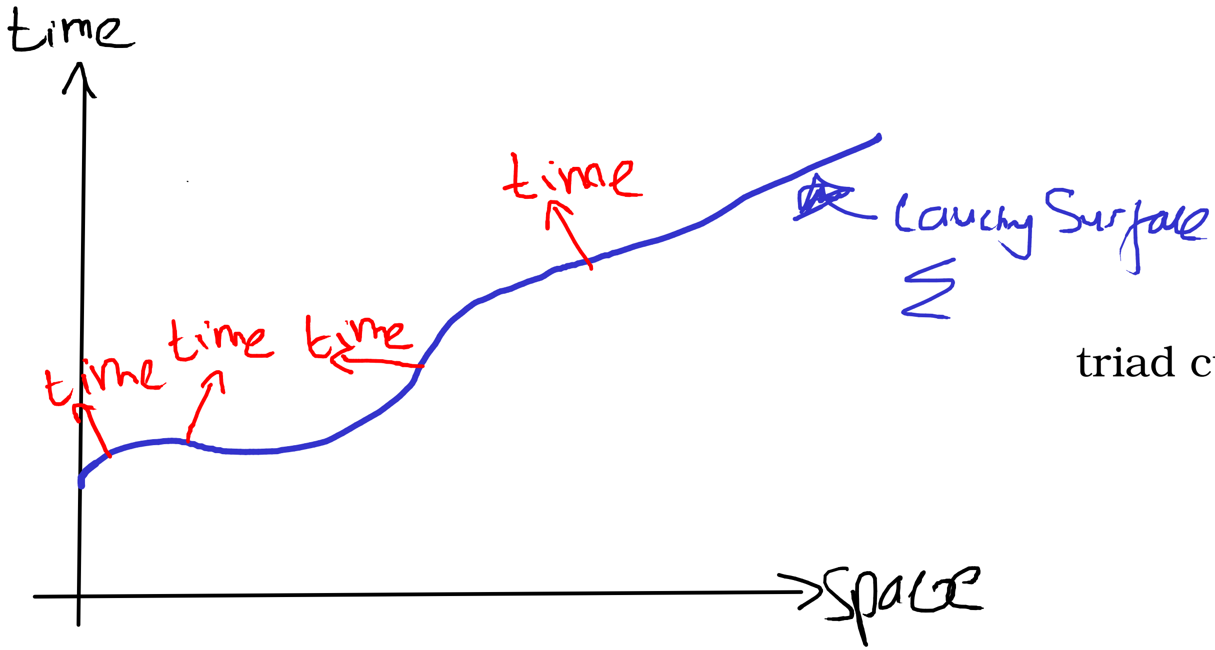
$$\boxed{\underline{K} = \gamma \underline{L}}$$

"magnetic" part of B

"electric" part of B

Abhay Ashtekar 1986

Ashtekar realised that the equations are easier to quantise if instead of considering 4D spacetime as a whole, one considers Cauchy surfaces evolving locally in some "time"



And define Ashtekar Field:

$$E^a_i(x) = \frac{1}{2}\epsilon_{ijk}\,\epsilon^{abc}\,e^j_b\,e^k_c$$

On Σ define triads, instead of tetrads.

$$q_{ab} = e^i_a(x)\,e^j_b(x)\,\delta_{ij}$$

triad curv. $\rightarrow K^a_i \, e^i_b = K^a_b$ ext curv

And Ashtekar connection:

$$A^i_a = \Gamma^i_a(e) + \beta K^i_a$$

Spin Connection

Sol^n of 3D Cartan eq^n

$$de^i + \epsilon^i_{jk}\,\Gamma^j \wedge e^k = 0$$

Now it is easier to deal with the Hamiltonian constraint of GR, with specific parameters, and it is found:

$$e \to B_f \in sl(2,\mathbb{C}) \text{ — algebra of}$$

$$\omega \to U_e \leftarrow \text{holonomies} = P e^{\int_e \omega} \in SL(2,\mathbb{C})$$

$$\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{C}^{2\times 2} : \det = 1 \right\}$$

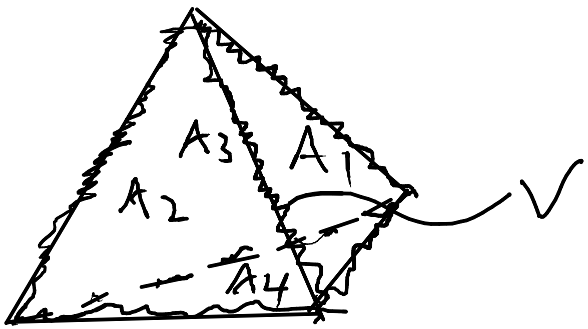
From these operators, eigenvalues of space are obtained. Specifically considering triangulated space, one gets 5 commutable operators

L_1, L_2, L_3, L_4, V with spectra:

$$L_n = 8\pi\hbar G\sqrt{j_n(j_n+1)} \leftarrow \text{areas}$$

volume $\to V_n$ (more complicated but discrete)

Fuzzy Tetrahedra



5 eigenvalues found, BUT, tetrahedron needs 6, so still has quantum uncertainty.

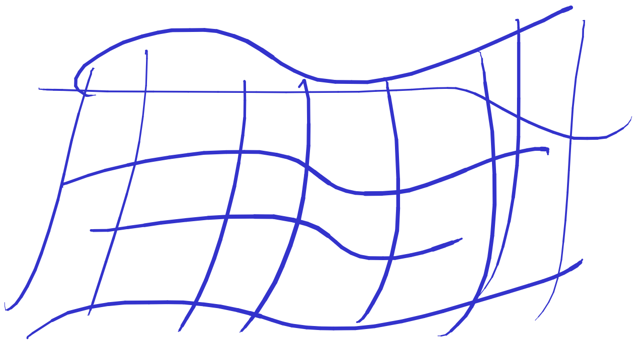
Tah-dah! The foundations of space!

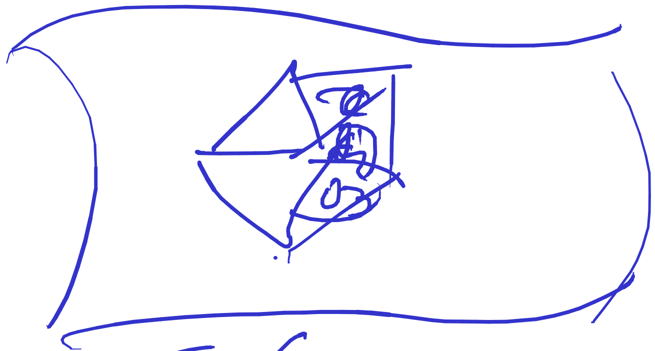
Regge Calculus

$(M, g) \longrightarrow (M, L_s)$

Riemannian

triangulation





$\delta_p = 2\pi - \sum \theta$

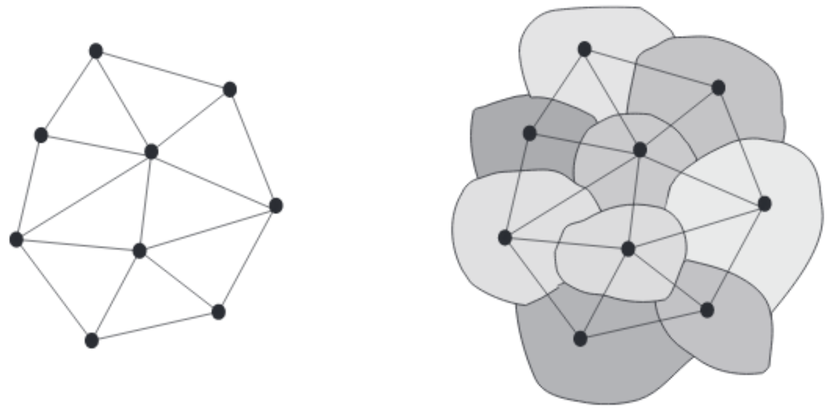
$$S(L_s) = \sum_p A_p(L_s)\, \delta_p(L_s)$$

(d-2) area

Tullio Regge

Spin Networks

Inspired by Penrose, but reused by Rovelli & Smolin



$|\Gamma, j_l, v_n\rangle$

FIG. 1. A spin network and the "quanta of space" it describes.

j are eigenvalues of representations of group elements and v's are intertwiners

*Zakopane Lectures, Rovelli arxiv 1102.3660

Spin Foams

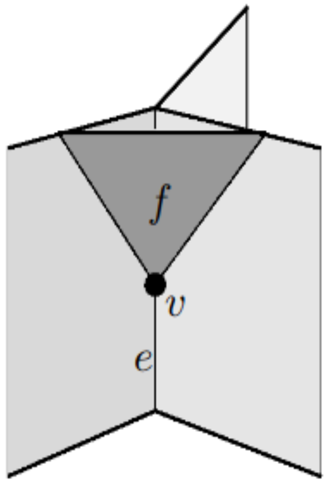
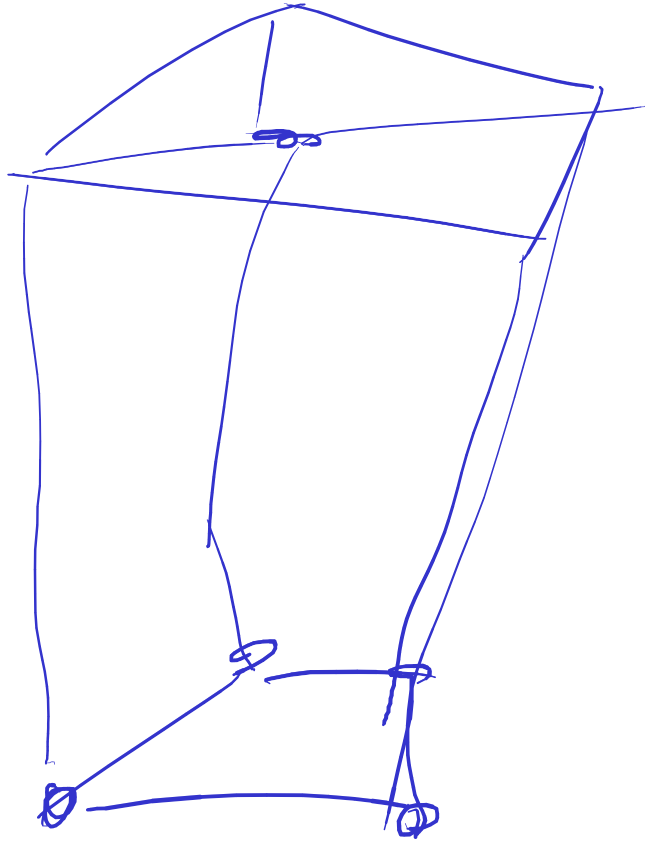
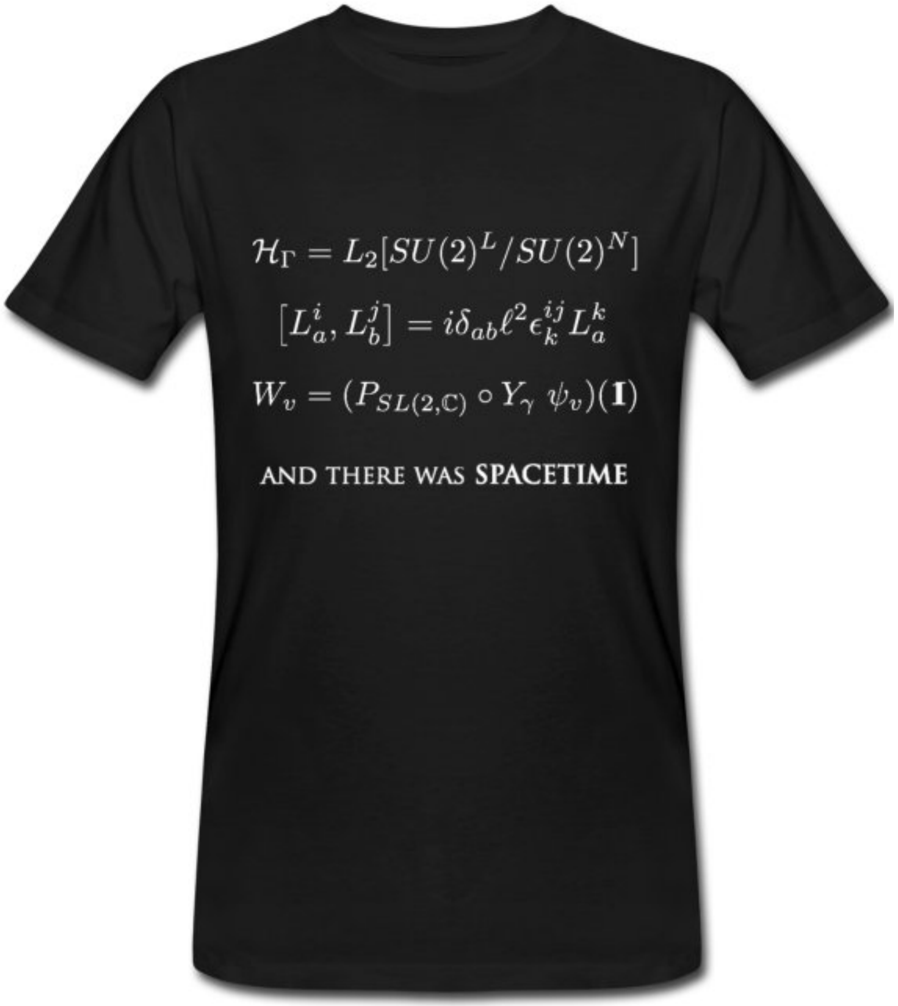


FIG. 7. A two-complex with one internal vertex.



EPRL Model



Quantum states of the geometry are described by functions $\psi(h_l)$ of elements $h_l \in SU(2)$ associated to the links l of an arbitrary graph Γ. Transition amplitudes between such states are defined perturbatively[1] by

$$Z_\mathcal{C}(h_l) = \int_{SU(2)} dh_{vf}\ \prod_f \delta(h_f)\ \prod_v A_v(h_{vf}). \qquad (1)$$

$\mathcal{C}$ is a two-complex (a combinatorial set of faces f that join along edges e, which in turn join on vertices v) bounded by Γ; $h_f = \prod_{v\in f} h_{vf}$ is the oriented product of the group elements around the face f and the vertex amplitude is

$$A_v(h_l) = \int_{SL(2,\mathbb{C})} dg'_e\ \prod_l K(h_l, g_{s_l} g_{t_l}^{-1}) \qquad (2)$$

where s_l and t_l are the source and target of the link l in the graph Γ_v that bounds the vertex v within the two-complex. The prime on dg_e indicates that one of the edge integrals is dropped (it is redundant). Finally, the kernel K is

$$K(h,g) = \sum_j \int_{SU(2)} dk\ d_j\ \chi^j(hk)\ \chi^{\gamma j,j}(kg). \qquad (3)$$

where $d_j = 2j+1$, $\chi^j(h)$ is the spin-j character of $SU(2)$ and $\chi^{\rho,n}(g)$ is the character of $SL(2,\mathbb{C})$ in the (ρ, n) unitary representation. γ is a dimensionless parameter that characterizes the quantum theory.

Seems to suggest that in semiclassical limit

Loop Quantum Cosmology

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

gets corrected by the factor

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) - \frac{k}{a^2} + \frac{\Lambda}{3}$$

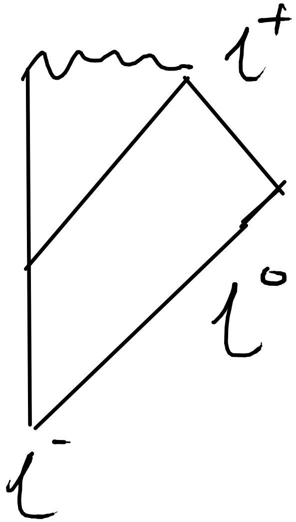
where the critical density is

$$\rho_c = \left(\frac{8\pi G}{3}\,\gamma^2 a_o\right)^{-1}.$$

$c = G = \hbar = 1$

Schwarzschild

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \left(1-\frac{2m}{r}\right)^{-1}dr^2 + r^2\, d\Omega^2$$



urgh!!! Problems: radiating light falls at

$$\dot{r} = 1 - \frac{2m}{r}$$

Silly

There has to be a better way!

Kruskal-Szekeres Metric

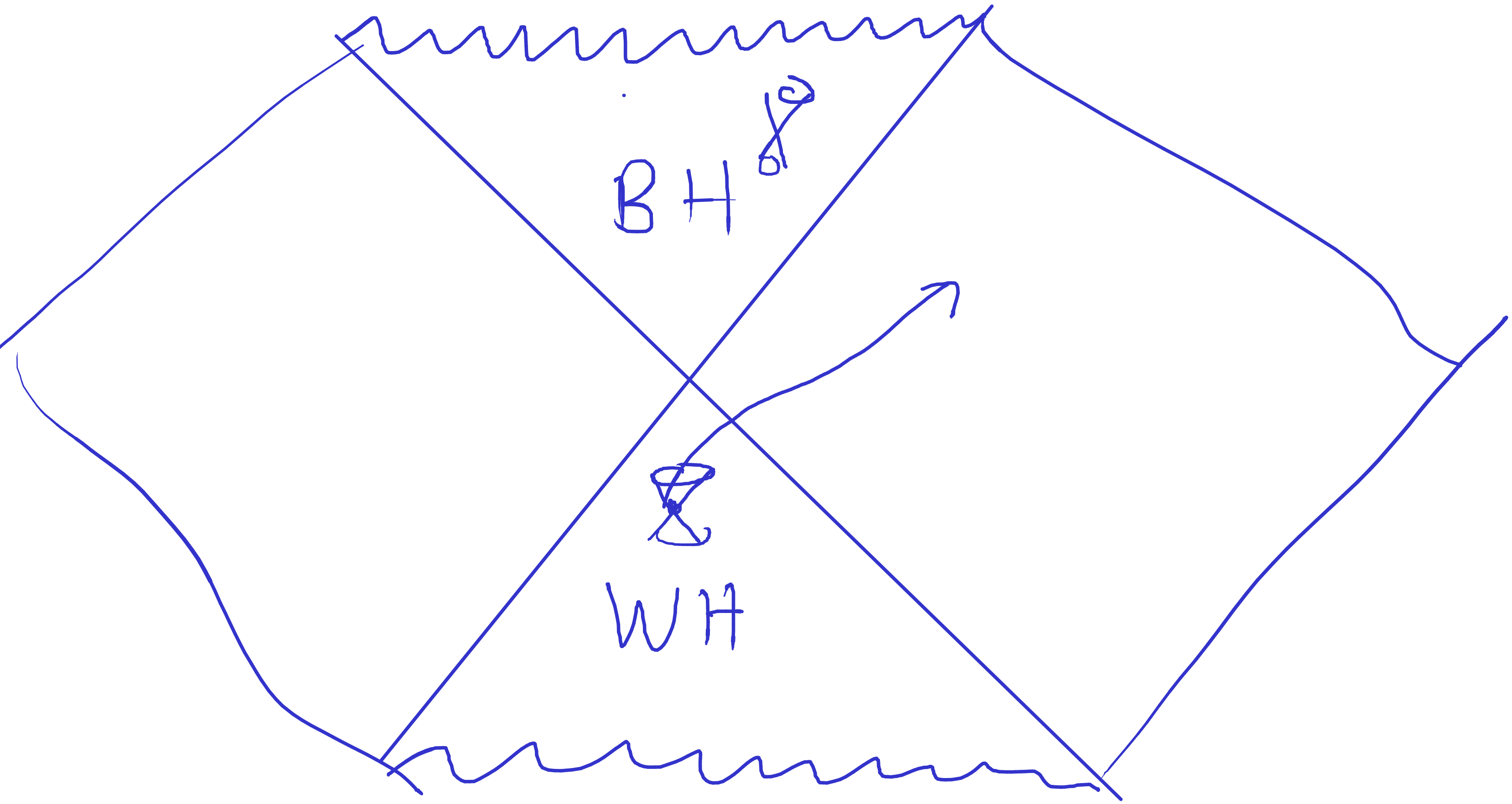
$$u = \sqrt{\left|\frac{r}{2m} - 1\right|}\, e^{\frac{r+t}{4m}} \qquad v = \sqrt{\left|\frac{r}{2m} - 1\right|}\, e^{\frac{r-t}{4m}} \operatorname{sgn}(2m-r)$$

$$ds^2 = -\frac{32m^3}{r} e^{-\frac{r}{2m}}\, du\, dv + r^2 d\Omega^2$$

Lambert fun^n / Product log

where $r(u,v) = 2m\left(1 + W\left(-\frac{uv}{e}\right)\right)$

Penrose Diagram



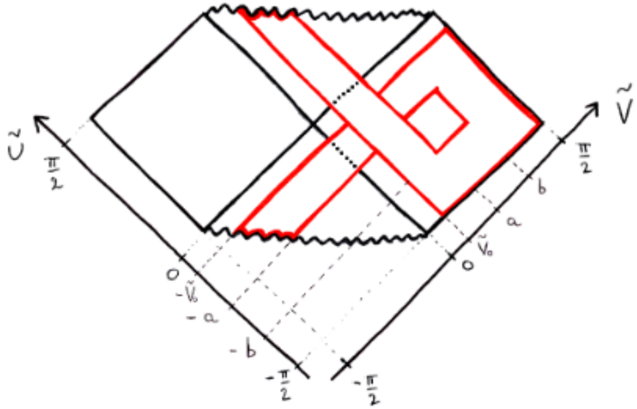


Figure 3: Penrose diagram of the Kruskal spacetime. The red straight lines are null, and the two red wavy lines will be identified after 'squashing the arms'. The inside region thus delimited is the spacetime of interest for us.

here be dragons!

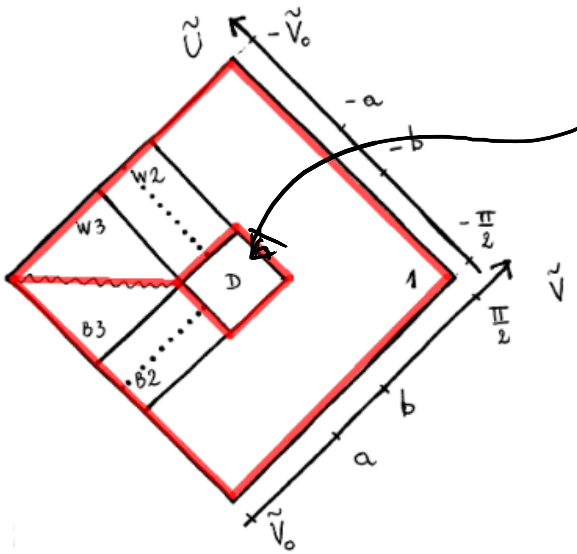
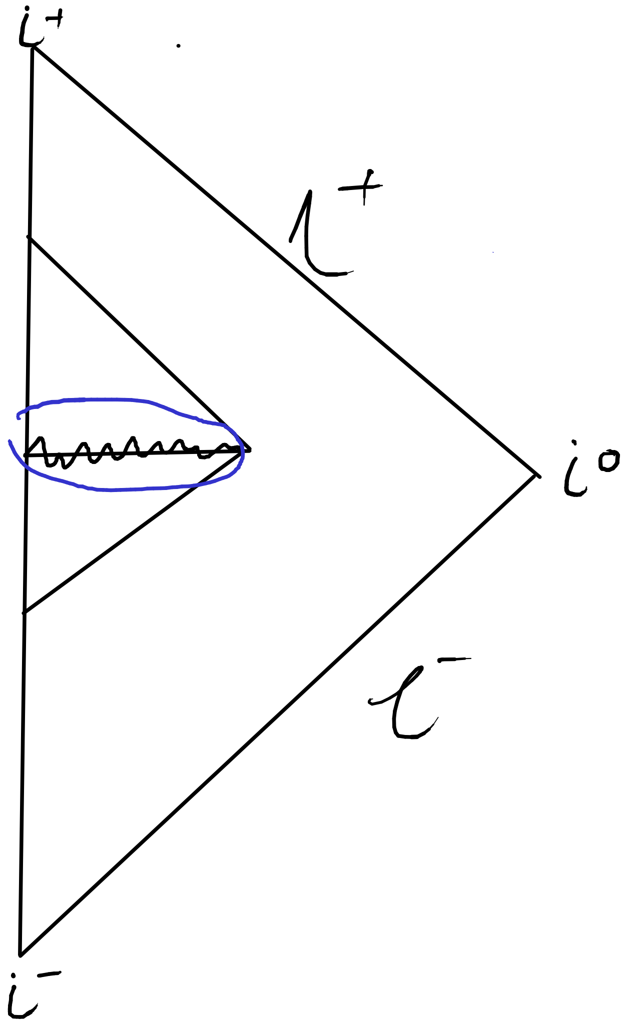
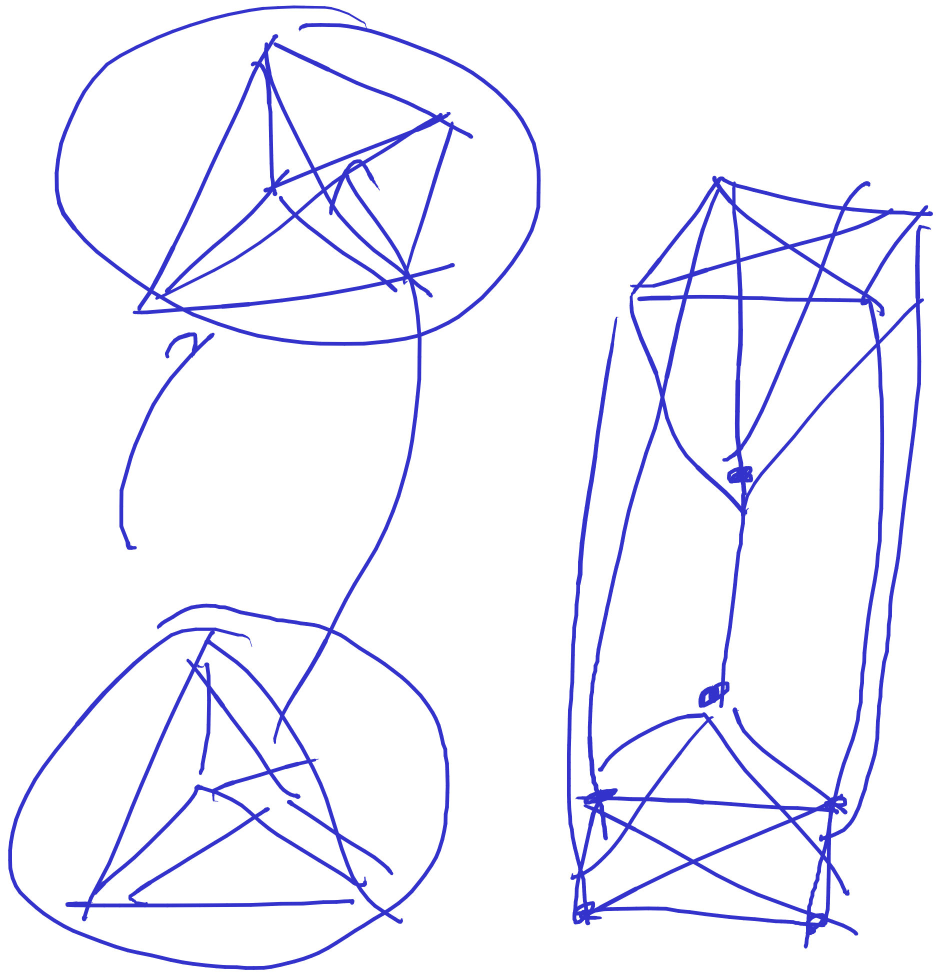


Figure 4: Penrose diagram of the outside of the null shell. The dotted lines are the two horizons at $r = 2m$.

Interior metric and ray tracing map, P. Martin-Dussaud, C. Rovelli arxiv 1803.06330

Transition





Timescale

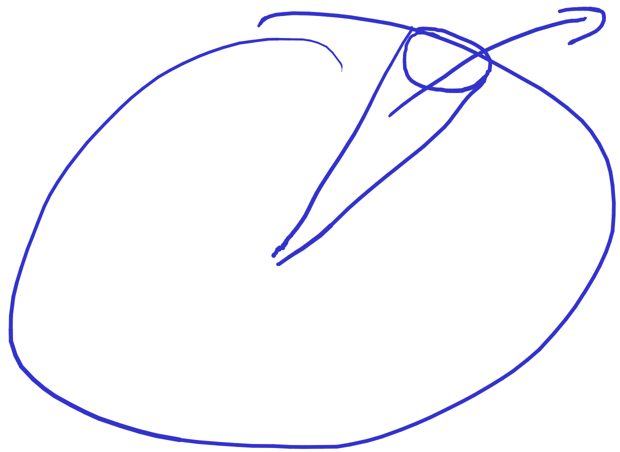
m^2

Planck Start Tunneling Time, Christodoulou. et. al., arxiv 1605.05268v3

e^m

Characteristic time scales, Christodoulou, d'Ambrosio, arxiv 1801.03027

What is the next approach?



$$ds^2 = -\frac{4(\tau^2 + l)^2}{2m - \tau^2} d\tau^2 + \frac{2m - \tau^2}{\tau^2 + l} dx^2 + (\tau^2 + l)^2 d\Omega^2$$

What could it mean/where could we find effects?

Благодаря