$F(R, T, X, \varphi)$ cosmology via Noether symmetry

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In recent years increased interest to the generalized theory of gravitation. This is result of the necessity to clarify the origin of the phenomenon of cosmic acceleration (dark energy), discovered by observing the radiation of SNe Ia. Observational data currently available is insufficient to determine a single model to explain the observed accelerated expansion of the universe. The most simple of all, it is proposed in due time by Einstein cosmological constant Λ. Currently, this model is reduced to the standard cosmological ΛCDM model.

Another example of the cosmological model describing the accelerated expansion of the steel model with a scalar field. Most popular of scalar field models is quintessence. Lagrangian density for this model $L = X - V(\varphi)$, were $V(\varphi)$ is function of scalar field φ, $X = \frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi$ is kinetic term of this field. As generalization of scalar field models used model of k - essence with Lagrangian density in most general form $L = F(\varphi, X)$ based on model of k-inflation.

Another example is geometrical approach like such simple gravity theory of gravitation as $F(R)$-gravity, which can be regarded as a generalization of general relativity. Here, instead of a Ricci scalar R is used function of Ricci scalar, which describes the presence of accelerated expansion. Also as another modified example of modify gravity can be represented teleparallel gravity $F(T)$, where T is a torsion scalar.

Thus, as we can see, popular study summarizing a variety of models, with a view to finding the most general properties of existing models and to determine the most relevant to observational data. For this reason for more generalization, we have the most general model in the form $F(R, T, X, \varphi)$, including scalar term with kinetic term. For the study of the model we use the Noether symmetry. This approach is widely used in various branches of physics, including cosmology. Here we consider the Friedmann - Lemaitre - Robertson - Walker metric with zero curvature.

From Noether symmetry equations we are have two possibilities. First is a linear equation:

$$F = s_1(\varphi)R + s_2(\varphi)T + s_3(\varphi)X. \tag{1}$$

Second we can recieve from homogeneous Monge - Ampere equations. After some calculations their solution involving arbitrary constants we can write as:

$$F = (C_1(\varphi)R + C_2(\varphi)T + C_3(\varphi)X)^2 + C_4(\varphi)R + C_5(\varphi)T + C_6(\varphi)X + C_7(\varphi). \tag{2}$$

This solution give us same results as recent observations about the early time inflation, close to R^2- Starobinsky model.

Solutions of Monge - Ampere equations involving one arbitrary function will give more general result:

$$F = f(C_1(\varphi)R + C_2(\varphi)T + C_3(\varphi)X, \varphi) + C_4(\varphi)R + C_5(\varphi)T + C_6(\varphi)X + C_7(\varphi) \tag{3}$$

For this model we can find several results. For example differentiating with respect to R with simplification $C_{1\varphi} = C_{2\varphi} = C_{3\varphi} = 0$, $\varphi = \varphi_0 t$ give solution as:

$$a = a_0 (t + t_0)^{c_0}. \tag{4}$$

For the case when the scalar field φ behaves independently of the scale factor a we can get for the scale factor the following solution:

$$a = a_0 e^{C_0 t}, \tag{5}$$

$$\varphi = C_8 t + C_9, \tag{6}$$

The same result can be obtained in cases where a u and v is proportional to the H or H^2.

We have considered the model with scalar field and $F(R)$, $F(T)$ - gravity in the most general form $F(R,T,X,\varphi)$ using the method of Noether symmetry. Also for this model received Euler-Lagrange equations. Also for this model received Euler-Lagrange equations. It is allowed to consider some special cases of model and receive scale factor.