Causality, locality, commutatively deformed general relativity, and dark matter:
A journey to the classical-quantum spacetime frontier and back

Paul G. N. de Vegvar

SWK Research
1438 Chuckanut Crest Dr., Bellingham, WA 98229, USA

Paul.deVegvar@post.harvard.edu

The contributions of Prof. Hermann Minkowski revolved around his radical unification of space and time into a four dimensional spacetime continuum, the geometrical nature of that union, and its Lorentz invariance. These ideas about classical flat spacetime, together with his emphasis on vital contact with experiment, still ring as true today as they did over a century ago. Variations on Minkowski's themes will run through this account of an exploration of the frontier between classical general relativity and canonical quantum gravity. Throughout the discussion conceptual issues, physical arguments and relationships to experiment and the standard model of particles are given priority over presenting mathematical details, which are available elsewhere.

This tale of journey and return includes two interconnected expeditions. In the first we inquire into the origins of classical causality and its relationship to locality. While the concept and consequences of local light cones are long established in general relativity, causality in canonical loop quantum gravity and in spin foam models has proven more difficult to elucidate. How do classical long range correlations of spacetime emerge from background independent non-pertubative theories of quantum gravity in some appropriate semiclassical limit? These correlations are then reflected in how one thinks about causality in quantum field theory on a fixed classical background spacetime, specifically that local observables at space-like separation commute (micro-causality). Generally the spin foam (covariant) models of the fully quantum regime do not support causal correlations, unless they are either assumed at the outset or involve some kind of alteration of the vertex amplitude intended to describe a local orientation. Approaching the issue of semiclassical causality from the canonical point of view is even more conceptually challenging since that approach is a "time-less" formalism. Consequently, the consensus expectation is that micro-causality will emerge from some as yet to be developed semiclassical limit of quantum gravity, however to date such a causal limit for quantum gravity remains lacking. Thus it is mysterious that micro-causality occupies such a foundational place in general relativity, quantum field theory, and the standard model of particle physics, yet is still so elusive from a background independent quantum gravity point of view. In a broader context, the semiclassical regime of quantum gravity is of importance not only from the perspective of causal correlations, but also as a general testing ground to examine whether a theory of quantum gravity can behave in familiar classical ways in some suitable limit, especially crucial given the conspicuous absence of experimental guideposts on the quantum side of the divide. Here we adopt the relational framework approach to canonical gravity which has been developed over several decades. We specifically explore the on-shell case where the constraints are non-ultralocal. Lieb-Robinson bounds, originally introduced in the 1970s to describe solid-state spin systems, are then applied to a spatially discretized version of the relational framework and describe how a suitably gauge invariant differential local light cone for discretized Dirac observables may be constructed. In essence, micro-causality emerges from on-shell *classical* non-ultralocality of constraints in a quite general sense within spatially discrete relational framework models with smooth monotonic gauge flow described by an external "time" parameter, even if the underlying quantum description of spacetime is acausal. When the constraints are taken to be ultralocal this Lieb-Robinson-based causal structure collapses. Spacetime remains classical in the sense that its gravitational degrees of freedom are not promoted to operators and their quantum fluctuations are ignored, while the matter fields on spacetime are quantum. Quantum spacetime fluctuations act to disrupt the micro-causal structure, and there is a set of general criteria which are sufficient for the Lieb-Robinson-based local light-cone structure to survive the quantum-classical tug-of-war.

However these methods of solid-state ancestry do not provide a microscopic origin for the classical spacetime nonlocalities, and all the four known interactions in Nature possess ultralocal Lagrangian densities. The second expedition of our journey bridges this chasm by utilizing Hopf algbra techniques on *commutatively* deformed 4-dimensional *curved* Lorentzian manifolds, where the nonlocal action of the (3+1)-diffeomorphism symmetries is described by Hopf algebras possessing a suitable Drinfeld twist. This simultaneously introduces a *commutative nonlocal* star product of objects living on a still classical manifold, meaning $f \star g = g \star f$. The deformed diffeomorphisms' nonlocal action on the physical fields differs from their pointwise action in undeformed classical general relativity, and those deformed symmetries obey a distinct Lie algebra, implying different physics. Aside from the commutative $\star$-product, it is sufficient to consider Hopf algebras with twists satisfying an Abelian constraint on their vector field generators. Imposing background independence requires those generators to be self-consistently related to matter fields. As a result it is found that the subtly deformed, but still classical, theory of spacetime naturally produces a nonlocality length ξ_c which can be $2-5$ orders longer than the Planck length L_P, and so spacetime acquires micro-causality at longer lengths via the Lieb-Robinson route. There is negligible experimental effect on the standard model of particles. Although baryonic twist producing matter would begin to behave acausally for rest masses above $\sim 1-10$ TeV, the other possibilities are viable dark matter candidates or a right-handed neutrino. In some cases twist generating matter has global or local continuous and discrete symmetries. First order deformed Maxwell equations imply an immeasurably small cosmological dispersion and produce a propagation horizon only for photons at or above Planck energies. Viewed as a whole, inquiring into the origins of micro-causality in the semiclassical frontier region surprisingly reveals a viable model for dark matter without appeal to grand unified theories, extra dimensions, supersymmetry, strings, mirror worlds, or modifications of Newtonian gravity. Simultaneously, field theory on classical spacetime becomes nonlocal.