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Title: The Mechanics of Spacetime – A Solid Mechanics Perspective on the Theory of General Relativity
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Short Abstract: We present an elastic constitutive model of gravity where we identify physical space with the mid-hypersurface of an elastic hyperplate called the “cosmic fabric” and spacetime with the fabric's world volume. Using a Lagrangian formulation, we show that the fabric's behavior, as derived from of Hooke's Law, is analogous to that of spacetime per the Field Equations of General Relativity. We relate properties of the fabric such as strain, stress, vibrations, and elastic moduli to properties of gravity and space (gravitational potential, gravitational acceleration, gravitational waves, and the density of vacuum). By introducing a mechanical analogy of General Relativity, we enable the application of Solid Mechanics tools to address problems in Cosmology and to give physical insight that was otherwise not present.

Extended Abstract:

We develop a formal analogy between Solid Mechanics and General Relativity by identifying physical space with the mid-hypersurface of a four dimensional hyperplate, called the Cosmic Fabric, which has a small thickness along the time dimension and exhibits a constitutive stress-strain behavior. Matter-energy fields cause the fabric to contract transversally, expand longitudinally, and consequently to bend. The effect is analogous to the result from General Relativity that matter causes space to bend resulting in gravity.

We show that the action integral of the cosmic fabric, $\mathcal{S}_{\mathrm{F}}$ below, when derived solely from Hooke's Law, is analogous to the Einstein-Hilbert action integral $\mathcal{S}_{\mathrm{EH}}$,

$$\mathcal{S}_{\mathrm{F}} = \int (\mathcal{L}_{\mathrm{B}} + \mathcal{L}_{\mathrm{M}})\, d^4x, \quad \mathcal{L}_{\mathrm{B}} = \frac{YL}{96c} R\sqrt{-\gamma}$$

$$\mathcal{S}_{\mathrm{EH}} = \int (\mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{M}})\, d^4x, \quad \mathcal{L}_{\mathrm{G}} = \frac{1}{2\kappa} R\sqrt{-\gamma}$$

where L is the reference thickness of the fabric, Y is its modulus of elasticity, γ is the determinant of the metric tensor, c is the speed of light, and R is the Ricci scalar. The terms $\mathcal{L}_{\mathrm{B}}$, $\mathcal{L}_{\mathrm{G}}$, and $\mathcal{L}_{\mathrm{M}}$ are Lagrangian densities due to, respectively, bending of the cosmic fabric, gravitational fields, and mass-energy fields. The action integral of any physical system fully determines its dynamics, because the system's governing differential equations can be derived by varying the action with respect to the spacetime metric. Therefore, once we recognize $\mathcal{S}_{\mathrm{F}}$ as analogous to $\mathcal{S}_{\mathrm{EH}}$, we can interpret various attributes of the cosmic fabric, such as its strain, stress, vibrations, and elastic moduli as analogous to properties of gravity and space (gravitational potential, gravitational acceleration, gravitational waves, and the density of vacuum).

The Cosmic Fabric model of gravity allows General Relativity problems to be formulated as Solid Mechanics problems, solved within the Solid Mechanics domain, and the solution interpreted back in General Relativity terms. The reverse is also true. Thus, ideas, methodologies and tools from one field become available to the other, and vice versa. Over the past century, Solid Mechanics and General Relativity have advanced independently from each other with only few researchers having expertise in both.

Consequently, there has been a significant terminology and focus gap between these two fields, which obscure their underling physical similarities. Our research attempts to bridge this gap.

We postulate the cosmic fabric to be a thin hyperplate, exhibiting isotropic hyperelastic constitutive behavior, with mass-energy fields as inclusions. First, we postulate that the fabric's world volume corresponds to the spacetime of General Relativity, and that the rate of the fabric's advancement in time for a given point along its mid-hypersurface is proportional to the fabric's thickness at that point. Second, the fabric exhibits isotropic hyperelastic constitutive behavior subject to Hooke's Law and the Poisson effect, which we extend to four-dimensions. Finally, we postulate that mass-energy fields behave as inclusions in the fabric inducing membrane strains leading to transverse displacement and hence bending to preserve mechanical compatibility. To keep the math tractable, we conduct our study under the simplifying assumption of weak and slow moving gravitational fields, and the assumption that fabric points along normals to the mid-hypersurface remain along the normals after deformation. We believe that these assumptions are not fundamental to the model, and that they could be changed in the future similarly to the advancements and changes in Solid Mechanics.

The Lagrangian density term $\mathcal{L}_\mathrm{B}$ is derived from considerations of the elastic bending energy. We treat the fabric as foliated into space-like hypersurfaces, and we average their elastic energy along normals to the mid-hypersurface, while treating the mid-hypersurface as neutral. We show that by choosing the fabric's material to have a Poisson ratio of $\nu = 1$, the resulting bending elastic energy at each point along the mid-hypersurface only depends, up to a constant, on the intrinsic three-dimensional curvature scalar $R^{3\mathrm{D}}$. Due to the Poisson effect, the intrinsic four-dimensional curvature R of the fabric's world volume outside of inclusions is coupled to $R^{3\mathrm{D}}$ and could be approximated as $R \approx 2R^{3\mathrm{D}}$. Finally, we arrive at the equation for $\mathcal{L}_\mathrm{B}$, stated above, in terms of the fabric's elastic modulus and reference thickness.

By identifying the bending Lagrangian density, $\mathcal{L}_\mathrm{B}$, with the gravity-induced Lagrangian density $\mathcal{L}_\mathrm{G}$, we could map various notions from Solid Mechanics to corresponding notions in General Relativity. We show that the fabric's time-like strain corresponds to the classical gravitational potential in the limit of weak and slow moving fields. Time dilation is proportional to the fabric's thickness. The inclusion stress, corresponding to the stress-energy of General Relativity, is associated with the energy required to deform the fabric to make room for an inclusion. Known materials with a Poisson's ratio of $\nu = 1$ have a fibrous substructure, which suggests that the cosmic fabric is, in fact, a fabric! Furthermore, $\nu = 1$ implies a vanishing p-wave modulus arises, meaning that only transverse waves are possible through the fabric. We show that transverse waves propagate at the speed of light; they correspond to gravitational waves, and have analogous polarizations. Because ordinarily longitudinal waves are faster than shear waves, one might ask why is the speed of light (shear wave) the fastest item in the universe. Answer: because $\nu = 1$, no longitudinal waves arise and so shear waves are the fastest. Using Planck's length as the fabric's thickness, we compute its elastic modulus, $Y = 8.8{\times}10^{113}\,\mathrm{N\,m^{-2}}$, and density, $\rho = 2.5{\times}10^{96}\,\mathrm{kg\,m^{-3}}$. The density agrees, to an order of magnitude, with the predictions of Quantum Field Theory for the density of vacuum.

Our research suggests an equivalence between postulating the field equations of General Relativity and postulating a cosmic fabric with material-like properties. We believe that these are two different approaches to studying the same underlying reality. The Cosmic Fabric model introduces a new paradigm for interpreting cosmological observations based on established ideas from Solid Mechanics. Recent advancements in Solid Mechanics have made possible the construction of multiscale models that accurately simulate the behavior of a broad range of materials and structures. The Cosmic Fabric model will enable the application of such techniques to simulate both the fine and large scale structures of the cosmos, and consequently, to address some of the outstanding problems in modern Cosmology.