Poisson-Minkowski spacetimes and Poincaré Drinfel'd doubles

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A possible group-theoretical approach to a quantum theory of gravity is based on the introduction of noncommutative spacetimes [1, 2] which are covariant under quantum group deformations of the isometry groups of Lorentzian spacetimes. From this perspective, the quantum group deformation parameter/s would be related to the Planck lenght or energy, and quantum groups [3, 4] are thought to play the role of the symmetries of the quantum spacetime, which is to be recovered when the deformation parameter/s go to zero. In this contribution we will concentrate in the case of vanishing cosmological constant (or vanishing curvature), for which the isometry group is the Poincaré group. Moreover, by working on a specially adapted kinematical basis we will be able to discuss in a more transparent way the physical interpretation of these noncommutative deformations.

The existence of a large amount of possible quantum deformations arises the question of which of them are more natural, and this leads us to consider the Drinfel'd double structure for the Poincaré Lie algebra $iso(3,1)$. In this contribution we will present the complete classification [5] of these Drinfel'd double structures in (1+1), (2+1) and (3+1) dimensions, and the result turns out to be strongly dimensional-dependent. In the (1+1) case two Drinfel'd double structures for the (nontrivially centrally extended) Poincaré algebra are found. In (2+1) dimensions, there exist six nonisomorphic Drinfel'd double structures for the Poincaré algebra, and it is proven that in (3+1) dimensions no Drinfel'd double structure exists. Each of the (1+1) and (2+1) Drinfel'd double structures here found generates a quantum Poincaré algebra whose properties will be analyzed.

Moreover, all these Drinfel'd double structures have -by construction- a canonical associated r-matrix (i.e. they are coboundary Lie groups) which endows the underlying Minkowski spacetime with an additional Poisson structure. These Poisson-Minkowski spacetimes are the semi-classical counterparts of the noncommutative Minkowski spacetimes arising from the corresponding quantum groups. By imposing the corresponding Poisson-Minkowski spacetimes to be Poisson homogeneous spaces with respect to the Poincaré group some of these deformations are naturally selected. We remark that this condition has not been frequently considered in the literature, albeit it is a quite natural one since it ensures that the corresponding quantum homogeneous space can be constructed.

The non-existence theorem for Drinfel'd double structures in (3+1) dimensions leads us to relax our conditions and consider more general deformations beyond Drinfel'd double ones [6]. In particular, we will present the well known κ-Minkowski spacetime associated to the κ-Poincaré group [7] and a twisted version of it, namely the twisted-κ-Minkowski spacetime [8]. In order to perform a more general selection for non-commutative spacetimes we will comment on work in progress on the characterization of Poisson homogeneous spaces in (3+1) dimensions. Moreover, we will sketch the generalization of this work to the case of non-vanishing cosmological constant, where the relevant groups of isometries are the de Sitter and anti-de Sitter ones [9, 10].

References

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