Hojman Symmetry Approach for Modified Chaplygin Gas Cosmological Model

Aizhan Myrzakul
Eurasian International Center of Theoretical Physics,
Eurasian National University, Astana 010008, Kazakhstan

ABSTRACT

In the present work we investigate the Hojman symmetry in FLRW cosmology. In particular, we use the Hojman symmetry to find conserved quantity of a particular cosmological gas model.

I. BRIEF REVIEW OF HOJMAN SYMMETRY

Let q_i be coordinates of some physical system. We assume that they satisfy the following set of second order ordinary differential equations [1]

$$\ddot{q}_i = F_i\left(q_j,\, \dot{q}_j,\, t\right), \tag{1}$$

where $i, j = 1, \ldots, N$ and a dot stands for a derivative with respect to time t, $F_i\left(q_j,\, \dot{q}_j,\, t\right)$ is a "force". Let q_i and $\tilde{q}_i$ be solutions of the same Eq. (1) (up to ϵ^2 terms) [**?**]. We assume that these solutions are related by the following infinitesimal transformation

$$\tilde{q}_i = q_i + \epsilon X_i\left(q_j,\, \dot{q}_j,\, t\right), \tag{2}$$

where $X_i = X_i\left(q_j,\, \dot{q}_j,\, t\right)$ is a symmetry vector for Eq.(1). It satisfies the following set of second order linear equations [2]

$$\frac{d^2 X_i}{dt^2} - \frac{\partial F_i}{\partial q_j} X_j - \frac{\partial F_i}{\partial \dot{q}_j}\frac{dX_j}{dt} = 0\,, \tag{3}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \dot{q}_i \frac{\partial}{\partial q_i} + F_i \frac{\partial}{\partial \dot{q}_i}\,. \tag{4}$$

Let the "force" F_i satisfy the equation (in some coordinate systems)

$$\frac{\partial F_i}{\partial \dot{q}_i} = 0\,. \tag{5}$$

Then the quantity

$$Q = \frac{\partial X_i}{\partial q_i} + \frac{\partial}{\partial \dot{q}_i}\left(\frac{dX_i}{dt}\right) \tag{6}$$

obeys the equation

$$\frac{dQ}{dt} = 0 \tag{7}$$

that is a conserved quantity for Eq.(1). Note that there exists one generalization of the last three equations. Instead of the equation (6), let the "force" F_i satisfy (in some coordinate systems) the generalized equation

$$\frac{\partial F_i}{\partial \dot{q}_i} = -\frac{d}{dt}\ln\gamma\,. \tag{8}$$

Here we assume that $\gamma = \gamma(q_i)$ is a function of q_i. In this case, the quantity Q takes the form

$$Q = \frac{1}{\gamma}\frac{\partial(\gamma X_i)}{\partial q_i} + \frac{\partial}{\partial \dot{q}_i}\left(\frac{dX_i}{dt}\right), \tag{9}$$

which is again a conserved quantity for Eq.(1). If $\gamma = const.$, then Eqs.(8) and (9) transform to Eqs.(5) and (6) respectively.

II. HOJMAN SYMMETRY IN MODIFIED CHAPLYGIN GAS

In this section, our aim is to find the Hojman symmetry for the Friedmann equations with the modified Chaplygin gas (MCG). The EoS of the MCG is given by

$$p = \gamma + \alpha\rho + \frac{\beta}{\rho^n}, \tag{10}$$

where γ, α, β are real constants. This formula can be rewritten as

$$p = \gamma + 3\alpha\dot{q}^2 + \frac{\beta}{3^n\dot{q}^{2n}}. \tag{11}$$

Then

$$\ddot{q}_1 \;=\; F(\dot{q}_1), \tag{12}$$

where

$$F(\dot{q}_1) = -\frac{1}{2}\left(\gamma + 3\alpha\dot{q}_1^2 + \frac{\beta}{3^n\dot{q}_1^{2n}}\right). \tag{13}$$

Following Egs.(3)-(9) the expressions for the scale factor and Hubble parameter are

$$a = a_0 e^{\left[\frac{n\beta}{3^n(3\alpha+\kappa)}\right]^{\frac{1}{2(n+1)}}t}, \tag{14}$$

$$H = \left[\frac{n\beta}{3^n(3\alpha+\kappa)}\right]^{\frac{1}{2(n+1)}} = const. \tag{15}$$

These expressions tell us that in this case the FLRW space-time turns to the de-Sitter one. The corresponding expressions for the density and pressure take the form

$$\rho = 3\left[\frac{n\beta}{3^n(3\alpha+\kappa)}\right]^{\frac{1}{n+1}}, \quad p = \eta + 3\alpha\left[\frac{n\beta}{3^n(3\alpha+\kappa)}\right]^{\frac{1}{n+1}} + \beta\left[\frac{n\beta}{3^n(3\alpha+\kappa)}\right]^{-\frac{n}{n+1}}. \tag{16}$$

The EoS parameter is

$$\omega = \frac{p}{\rho} = \frac{\eta}{3}\left[\frac{n\beta}{3^n(3\alpha+\kappa)}\right]^{-\frac{1}{n+1}} + \alpha + 3^{-(n+1)}\beta\left[\frac{n\beta}{3^n(3\alpha+\kappa)}\right]^{-1}. \tag{17}$$

Let us now find X_1. The solution of the equation (3) for X_1 we look for as $X_1 = \epsilon\dot{q}_1$ [3].

[1] S. Hojman, J. Phys. A: Math. Gen. **25**, L291 (1992).
[2] R. M. Santilli, *Foundations of Theoretical Mechanics I*, Springer, New York (1978).
[3] A. Myrzakul, R. Myrzakulov, *On the Hojman conservation quantities in FRW Cosmology*, [arXiv:1603.01611].