Classical and Quantum Singularities in "Cut-and-Pasted" Minkowski Spacetime

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ABSTRACT:

"Cut-and-Pasted" Minkowski spacetime in 4 and higher dimensions holds a wealth of structure. That structure and the incumbent singularities will be reviewed here in 4 dimensions, and new results given for higher dimensions. The classical singularities in these flat spacetimes are termed quasiregular in the usual classification and are necessarily topological in nature.

A quasiregular spacetime is a spacetime with a quasiregular singularity. The 2-dimensional cone has a quasiregular singularity at its apex; its 4-dimensional analog, the idealized cosmic string spacetime, is also quasiregular. An n-dimensional cosmic string spacetime, as will be discussed here, is also quasiregular. All of these examples have *disclinations* in the language of distortions.

The language of distortions, first used to describe elastic media, including solid continua and crystals, has relatively recently been adapted to describe simple quasiregular spacetimes. There are two types of distortions, disclinations and dislocations, and two types of dislocations, screw and edge. A complete classification of flat spacetime distortions is given by Puntigam and Soleng (1997), based on the symmetries of flat spacetime.

In three-dimensional elastic media a process known as the Volterra process uses the $SO(3) \otimes T(3)$ symmetries of flat space to classify the distortions of flat space; 3 disclinations are related to the $SO(3)$ symmetry and 3 dislocations are related to the $T(3)$ symmetry. In four-dimensional Minkowski spacetime a generalized Volterra process gives 10 differently structured Riemann-Cartan spacetimes based on the $SO(1,3) \otimes T(4)$ symmetry of Minkowski spacetime; 6 disclinations are related to the $SO(1,3)$ symmetry and 4 dislocations are related to the $T(4)$ symmetry. All of these distortions can be made by "cutting and pasting" Minkowski spacetime.

A particularly interesting spacetime with one disclination and two screw dislocations has been discussed by Gal'tsov and Letelier (1993) and by Tod (1994). This Gal'tsov-Letelier-Tod "GLT" spacetime is described by the metric,

$$ds^2 = -(dt + \alpha d\phi)^2 + dr^2 + \beta^2 r^2 d\phi^2 + (dz + \gamma d\phi)^2,$$

with α, β, γ constants and the usual coordinate ranges. It is a flat spacetime with closed timelike lines if $\alpha \neq 0$. As Tod explains, if $\alpha = 0, \gamma = 0$, and $\beta^2 \neq 1$, this metric describes the idealized cosmic string; if $\alpha = 0$ and the final term $(dz + \gamma d\phi)^2$ is missing, this is the 'point source" of 2+1 gravity; and, if $\alpha = 0, \gamma \neq 0$, this is the asymptotic metric at large spatial separation from a cylindrically-symmetric gravitational wave. In this talk I am solely interested in the static version of this metric, without any timelike distortions and associated closed timelike lines.

This talk will focus on the usual idealized 4-dimensional cosmic string, its generalization with a spacelike distortion, and its n-dimensional generalization.

Consider, in particular, the zero-thickness n-dimensional idealized cosmic string. It has a metric,

$$ds^2 = -dt^2 + dr^2 + \beta^2 r^2 d\phi^2 + dz^2 + \sum_{i=5}^{n} dw_i^2,$$

where $-\infty < t, z, w_i < \infty$, $0 < r < \infty, 0 < \phi < 2\pi$, and $\beta^2 \neq 1$. This spacetime has incomplete, inextendible geodesics that run into the quasiregular singularity at $r = 0$, where there is a δ-function in curvature. In the language of distortions, $r = 0$, is a *disclination*; this classical singularity is indicated by nontrivial holonomy. Is it quantum mechanically singular as well? This is a question we will attempt to answer.

To decide whether a spacetime is quantum mechanically singular we will use a criterion introduced by Horowitz and Marolf (1995), following early work by Wald (1980). They call a spacetime quantum mechanically nonsingular if the evolution of a test wave packet in the spacetime is uniquely determined by the initial wave packet, the manifold, and the metric alone, without having to place arbitrary boundary conditions at the classical singularity. Technically, a static or conformally-static spacetime is quantum mechanically singular if the spatial portion of the wave operator is not essentially self-adjoint.

T.M. Helliwell, G.C. Dietzen and myself have studied scalar wave behavior on the n-dimensional idealized cosmic string as follow-on work on the 4-dimensional static GLT spacetime and have found wave modes that leave the classical quasiregular singularity intact as well as modes that erase or heal the singularity. This work will be described in this talk along with the necessary mathematical background.