Alternatives to General Relativity and the Nature of Gravitation

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Extended Abstract

Attempts to emulate Einstein's theory of spacetime and gravitation are in fact as old as the theory itself. Although many such "alternative gravity" theories have been ruled out by experiment over the years, still others have emerged and overall there appears to be considerable room left for alternatives to general relativity at the time of writing, if attention is restricted mainly to the experimental state of affairs (examples include the ECKS torsion theory, $f(R)$-models, GEA-models, massive gravity, TeVeS-theory and conformal gravity, to name just a few). As it happens, except for the first example, all alternatives just mentioned belong to the class of so-called *metric theories* of gravity, which are theories that incorporate what is usually referred to as the *Einstein Equivalence Principle* (EEP). Since the empirical evidence for the validity of EEP is very strong, it is often argued that, with a few exceptions (such as the mentioned ECKS torsion theory, for which the violation of EEP would be too minute to possibly detect with current technology), the only alternative gravity theories that have a hope of being viable are metric theories [1].

I will review this line of argument, first by presenting an overview of various formulations of the Equivalence Principle and their interconnectedness - in particular, two versions respectively weaker (WEP) and stronger (SEP) than EEP - and second, by presenting a detailed derivation of what EEP can be said to imply. A particular aspect of this second point is that of *universal coupling*, i.e., the property that all matter fields (or "non-gravitational" fields more generally) couple in an identical fashion to a *single* gravitational field, namely the spacetime metric, g_{ab}. In general relativity, which is the only (experimentally viable) metric theory known to satisfy SEP, this exhausts the gravitational sector. However, other metric theories of gravity - such as the alternatives to general relativity mentioned earlier - include additional "gravitational" fields in their formulation, which may either be dynamical or not ("background geometry").

This gives rise to some important foundational questions. In particular, since in accordance with metricity, g_{ab} is the only gravitational field that couples to matter fields (or "non-gravitational" fields more generally), it may be asked in which sense - other than semantic - these additional fields are precisely "gravitational". This is probably best illustrated in an action based formulation. Denoting by Ψ and Φ respectively any "gravitational" fields other than g and any "non-gravitational" fields (such as the Yang-Mills, spinor and Higgs fields figuring in the particle physics standard model, for instance), the total action is of the form

$$S[g, \Psi, \Phi] \;:=\; S_{\text{grav}}[g, \Psi] \;+\; S_{\text{non-grav}}[g, \Phi] \tag{1}$$

with the two terms on the right-hand side prescribing the dynamics of the "gravitational" and "non-gravitational" sectors, respectively (this incidentally also means that some formulations of specific theories, such as the so-called Einstein frame in scalar-tensor gravity, are considered to be non-physical). The only possible role for the Ψ-fields is that of potential "sources", together with the Φ-fields, for the metric field. Based on this, together with the fact that there is no coupling between Ψ and "ordinary matter", it would appear that one could just as well refer to the Ψ-fields as "dark matter" fields that just so happen to couple to the gravitational field in a rather peculiar way (i.e., through their appearance in the part of the action that partially determines the g-dynamics). As will be demonstrated in detail, these remarks are further corroborated by the fact that some of the mentioned "alternative gravity" theories appear to allow for stress-energy

tensors for the Ψ fields (and would therefore be in conflict with the usual relativistic credo that, by virtue of the equivalence principle, there is no meaningful local notion of gravitational energy). Finally, in some cases it is not clear that metricity of the examples listed is actually interpretation-independent. For instance, Bekenstein's Tensor-Vector-Scalar (TeVeS) theory [2] was developed explicitly as a relativistic version of Milgrom's phenomenological MOND model [3]. The latter however is often said to allow for two possible physical interpretations (that is, either as "modified gravity" or as "modified inertia"), whereas conflicting statements can be found in the literature as to which of these two versions satisfies (some relevant version of) the equivalence principle.

References

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