The Role of Time for Reparametrization-Invariant Systems

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We discuss reparametrization-invariant systems, mainly the relativistic particle and its D-dimensional extended-object generalization to d-branes. For a d-brane that doesn't alter the background fields, we define non-relativistic equations assuming integral sub-manifold embedding of the d-brane. We argue for a one-time-physics as an essential ingredient for a non-relativistic limit. The mass-shell constraint and the Klein-Gordon equation are shown to be universal when gravity-like interaction is present. Our approach to the Dirac equation follows Rund's technique for the algebra of the γ-matrices that doesn't rely on the Klein-Gordon equation [1].

We discuss some aspects of the non-relativistic, relativistic, and a la Dirac-equation quantization of reparametrization-invariant systems. In its canonical form, the matter Lagrangian for reparametrization-invariant systems contains well known interaction terms, such as electromagnetism and gravity. For a reparametrization-invariant systems there are constraints among the equations of motion, which is a problem when attempting to quantize such system. Nevertheless, there are procedures for quantizing such theories [1, 2, 3, 4, 5]. Here, we will demonstrate another approach ($v \to \gamma$) that takes advantage of the fact that the corresponding Hamiltonian is identically zero ($H \equiv 0$) for reparametrization-invariant systems.

Furthermore, we argue that a one-time-physics is needed to assure causality via finite propagational speed in case of point particles. For d-branes the one-time-physics reflects separation of the internal from the external coordinates when the d-brane is considered as a sub-manifold of the target space manifold M. The non-relativistic limit is considered to be the case when the d-brane is embedded as a sub-manifold of M.

Some arguments for 4D space-time are based on geometric and differential structure of various brane and target spaces [6, 7]. All these are reasons why the spacetime seems to be four dimensional. Here we present an argument that only one-time-physics is consistent with a finite propagational speed. Thus, resulting in 1+3 Minkowski space-time.

In summary, we discuss the structure of the matter Lagrangian (L) for ex-

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tended objects. Imposing reparametrization invariance of the action S naturally leads to a first order homogeneous Lagrangian. In its canonical form, L contains electromagnetic and gravitational interactions, as well as interactions that are not clearly identified yet.

The non-relativistic limit for a d-brane has been defined as those coordinates where the brane is an integral sub-manifold of the target space. This gauge can be used to remove reparametrization invariance of the action S and make the Hamiltonian function suitable for canonical quantization. For the 0-brane (the relativistic particle), this also has a clear physical interpretation associated with localization and finite propagational speed.

The existence of a mass-shell constraint is universal. It is essentially due to the gravitational (quadratic in velocities) type interaction in the Lagrangian and leads to a Klein-Gordon equation. Although the Klein-Gordon equation can be defined, it is not the only way to introduce the algebra of the γ-matrices needed for the Dirac equation. The algebraic properties of the γ-matrices may be derived using the Lie group structure of the coordinate bundle; these properties are closely related to the corresponding metric tensor $g^{\alpha\beta} = \left\{\gamma^\alpha, \gamma^\beta\right\}$ and may restrict the number of terms in the Lagrangian. Once the algebraic properties of the γ-matrices are defined, one can use $v \to \gamma$ quantization in the Hamiltonian function $H = pv - L(x, v)$ to obtain the Dirac equation.

References

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