Rigid body motion in special relativity

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We study the acceleration and collisions of rigid bodies in special relativity. We first show that the definition of 'rigid body' in relativity differs from the usual classical definition, so there is no difficulty in dealing with rigid bodies in relativistic motion. In classical (prerelativistic) dynamics, the motion of a rigid body is generally defined as preserving the dimensions of the body during any motion of the body. However, the proper relativistic definition of a rigid body turns this classical definition on its head. If an object retained its length while moving, its length would increase in its rest system. Consequently, we take as our definition of a rigid body that *a rigid body retains its rest frame length while in motion.* This requires a moving rigid body to change its length in any frame in which it is moving.

We consider the motion of a rigid rod of length L_0 that starts from rest in a Lorentz system S, and accelerates at constant acceleration until every point on the rod is moving with a velocity V. Each point on the rod undergoes a constant acceleration in its instantaneous rest system S$'$, but different points on the rod have different accelerations that are related by the equation

$$x - x_B = \frac{1}{a'(x')} - \frac{1}{a'_B}. \tag{1}$$

Here $x - x_B$ is the distance from the back end of the rod to a point x on the rod in the system S in which the rod is moving. x' is the rest frame distance from the back end of the rod, and a'_B is the acceleration of the back end.

We see that in order to keep a body rigid in its rest frame, the acceleration has to vary throughout the body in a specific way. In an accelerating spaceship, passengers in the front-end would feel less push on their backs

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than passengers in the rear end of the ship. The spatial variation in the acceleration has the effect of producing the Lorentz contraction when all points on the rod have reached the final constant velocity V, but the passengers in the spaceship would observe no change in any of its dimensions.

The motion of the rod in frame S is shown in as the solid trajectory in Fig. 1.

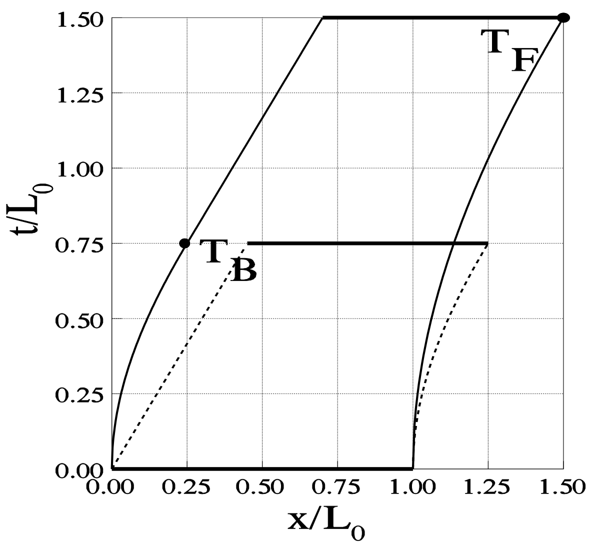

Fig.1: Constant acceleration of a rigid body. The solid curve is the trajectory for continuous acceleration. The figure represents the space-time curve for acceleration in frame S from rest to a final velocity $V = 0.6$, for which $\gamma = 1.25$. The rest frame accelerations for the front and back ends of the rod are $a'_F = 1/(2L_0)$ and $a'_B = 1/L_0$, which are consistent with Eq. (1). The time $\mathbf{T_B}$ on the solid curve represents the end of acceleration for the back end of the rod, and $\mathbf{T_F}$ for the front end. The acceleration continues until each end of the rod reaches velocity V, which occurs at equal times in the rest system, but at the unequal times $\mathbf{T_F}$ and $\mathbf{T_B}$ shown on the figure. The dashed curve is for impulsive acceleration that occurs when any part of the rod is struck with an impulsive force, in which case the back end acceleration is instantaneously infinite, so $1/a'_B = 0$.