Extended abstract

Operational existence of a spacetime manifold

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Introduction

There is a long-standing argument that the individual points of a spacetime manifold are physically unobservable, given the principle of general relativity, i.e. because we expect all physics to be diffeomorphism-invariant and background-independent. As far as the statement goes, one cannot distingush between "this point" versus "that point" of spacetime itself, but only "the point where fields have this value" versus "the point where fields have that value". While the original idea historically dates back to the relationism/substantivalism debate between Newton and Leibniz (or possibly even further back), the modern formulation of the relational nature of physics was nicely phrased by Carlo Rovelli[1]:

> *The world is made up of fields. Physically, these do not live on spacetime. They live, so to say, on one another. No more fields on spacetime, just fields on fields.*

Simply put, the basic argument against the observability of spacetime points goes as follows. If we choose one point of spacetime (by specifying its coordinates in some coordinate system), observe the values of all fields at that point, and then perform an "active diffeomorphism" (permutation of manifold points), we "move" all physics from that point to another point. After that, we can perform a "passive diffeomorphism" (choice of a different manifold chart), to undo the active one, i.e. we use the same set of numbers as coordinates for the new point in the new coordinate chart as we have used for the old point in old coordinates. Given that physics does not change throughout the process, we conclude that one cannot distinguish between the "old spacetime point" and the "new spacetime point". Thus spacetime points are unobservable.

While all this is correct, one is often tempted to make a more general claim that the whole spacetime manifold is an unobservable entity, given that its individual points cannot be observed. *The purpose of this lecture is to scrutinize that claim.* Namely, individual points aside, the spacetime manifold as a global entity has additional properties, which are simultaneously both observable and diffeomorphism-invariant. Such properties are the manifold's dimension and topology. Thus, we argue that observing the dimension and topology of the spacetime manifold could grant it *objective, physical existence*, despite unobservability of its individual points. This stands in sharp contrast with the relational ideas that ultimately there is no spacetime and that "fields live on fields".

[1] C. Rovelli, *Quantum Gravity*, Cambridge University Press, Cambridge 2004.

The strategy

We aim to address the following two questions:

(a) Given some model-independent theoretical framework, can the dimension and topology of the spacetime manifold actually be observed? If so, what would be the exact operational experimental protocol to do this? How generic is this procotol, i.e., how much is it model-independent?

(b) Assuming the answer to (a) is affirmative, how does any detailed relational model of physics account for the measurement of the dimension and topology of spacetime, without assuming spacetime to begin with? Or more simply put, can one deduce from the theory that spacetime is 4-dimensional and simply-connected?

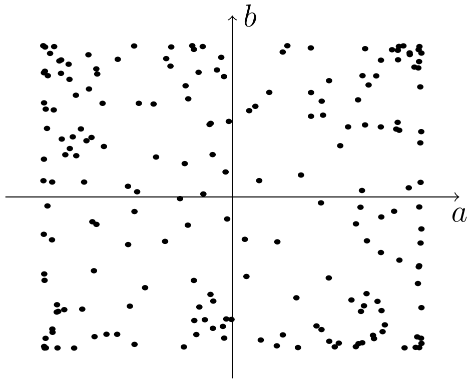
The question (a) challenges the substantival point of view — if one wants to claim that spacetime indeed *does have* objective reality, because one can consistently measure its dimension and topology, one must first demonstrate that such measurements are possible, and that they give an unambiguous result, using a model-independent theoretical framework.

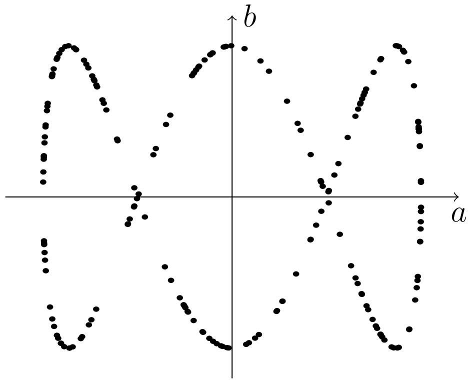
The question (b) challenges the relational point of view — if one wants to claim that spacetime *does not have* objective reality, one must give a theoretical prediction for the result of the experiment established in (a), i.e., derive the dimension of spacetime from first principles of some proposed relational theory which does not a priori postulate any spacetime manifold. This kind of prediction has never been explicitly performed in any relational theory of physics.

The method

Modern theoretical description of physics is built on an intuitive assumption that we live in a spacetime where physical entities have length, width, height and age, giving rise to the dimension of the spacetime manifold being $D = 4$. All experiments and observations ever performed in the history of science so far support this conclusion, over an impressive range of scales — from the scale of 10^{26} m (the size of the observable Universe) down to the scale of 10^{-20} m (the distances probed by the current LHC and LIGO experiments). Outside of these 46 orders of magnitude, one can in principle hope to observe an additional 15 orders of magnitude, from 10^{-20} m all the way to 10^{-35} m (the Planck scale). The latter range is outside of our current technological capabilities, so we do not have any data to either support or falsify the claim that $D = 4$.

Of course, one can ask the question how do we actually know that the dimension of the physical spacetime is equal to 4 in the so far observed range of scales. To that end, we propose a method (a gedanken-experiment) to observe the dimension and topology of a manifold, which can be illustrated on a couple of toy-examples as follows. Given two observables, A and B, we can perform multiple measurements of each, and obtain multiple results, a_i, b_i (where $i = 1, \dots, N$). We can then plot those results as points in the a-b configuration space. If the observables A and B are not correlated, the data will be scattered all over the plot, which will look like the diagram on the left:



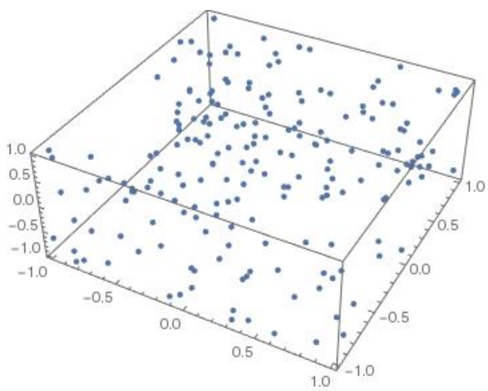


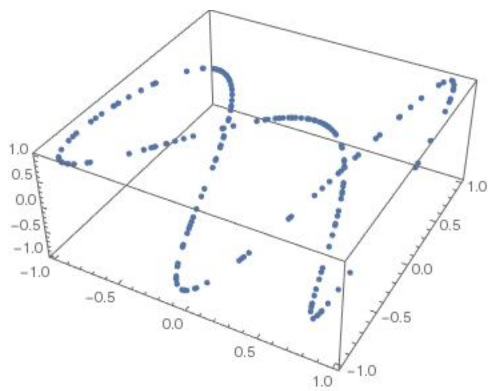
However, it may happen (as it does in the real world) that there exists a particular type of correlation between A and B, such that the plot looks like the diagram on the right, revealing the underlying structure which defines the correlation — in the example above, a 1-dimensional closed curve, topologically equivalent to a circle S_1 (up to self-intersections). In this case, one can parametrize the points on the curve with a parameter $t \in [0, 2\pi)$, and encode the correlation between the observables A and B into a statement that they both depend on t, giving rise to parametric equations for the measurement outcomes:

$$a = A(t)\,, \qquad b = B(t)\,.$$

In this way, the correlations between observables provide us with an insight that these observables are actually two fields "living" on a 1-dimensional manifold S_1. Note that changing the parametrization from t to t' does not change anything in the diagram, which means that the correlation between the two fields is invariant with respect to 1-dimensional diffeomorphisms of S_1.

Moreover, we can extend our analysis to include another, third observable, C. If the observables are uncorrelated, the plot of the measurement results of all three observables in the a-b-c configuration space will resemble the $3D$ diagram on the left:





However, there may exist a suitable correlation between the observables, so that instead we obtain the diagram on the right, again revealing the underlying manifold S_1, which allows us to write the parametric equations

$$a = A(t)\,, \qquad b = B(t)\,, \qquad c = C(t)\,,$$

and to interpret the observables A, B and C as fields living on a 1-dimensional circle. The fact that the diagram on the right reveals a circle is an *intrinsic property of the observables*, since in principle we could have obtained a different plot, where the points (a_i, b_i, c_i) were arranged on a 2-dimensional sphere instead of a 1-dimensional circle.

Using the kind of reasoning illustrated in the above toy-examples, we argue that in a similar way one can infer that the proper observables from the real world always display correlations reflecting an underlying structure of a 4-dimensional manifold $\mathcal{M}_4$, which we call spacetime, and that its topology is simply-connected on the scales that can be tested. Specifically, regardless of the choice and the number K of observables we are sampling, it somehow magically turns out that they always display just the right amount of correlations, so that one can write them in the form of parametric equations

$$a_1 = A_1(t,x,y,z)\,, \qquad \dots, \qquad a_K = A_K(t,x,y,z)\,,$$

where t, x, y, z are parameters of some chart on some manifold $\mathcal{M}_4$. In that sense, we can say that the observables $A_1, \dots, A_K$ are fields living on $\mathcal{M}_4$. Note that this conclusion is diffeomorphism-invariant, since the choice of a different parametrization t', x', y', z' instead of t, x, y, z does not in fact change anything in the correlation diagram. The fact that we obtain $D = 4$ is an *intrinsic experimental property of the observed outcomes*, since in principle we could have obtained a correlation that corresponds to $D = 3$ or $D = 5$ or otherwise.

Concluding remarks

Of course, despite the fact that the diagrams above appear suggestive, it is far from obvious how one can deduce the existence of a manifold and its dimension and topology, based on a scatter-plot of individual data points, in the general case. Therefore, we will describe a more rigorous and complete mathematical technique that performs both of these tasks.

In light of the above, the question (b) becomes highly nontrivial. Namely, given a theoretical model which describes the detailed dynamics of observables $A_1, \dots, A_K$, and which explicitly *does not assume* the existence of an underlying spacetime manifold (in line with the relational point of view), the challenge is to *deduce from theory* that there exist some very peculiar correlations between the observables, describing their dynamics *as if they were fields living on a manifold* with a specific dimension and topology. In particular, a viable realistic theoretical model ought to give rise to the result $D = 4$ purely from the interactions between the fields in the model. This would amount to a notion of "emergence" of spacetime. However, so far no such model has ever been constructed, and until one is, we can argue that spacetime is a notion that objectively exists in its own right, as part of our physical reality.