Proof of the Relativity Principle

7.15.2022 Hiroaki Fujimori

The Problem left by Poincaré

Looking back on the history of the special relativity, Lorentz and Poincaré were on their way to give a mathematical proof to the results of the Michelson-Morley experiment. Meanwhile Einstein published the theory of relativity based on the principle of relativity and the constancy of the speed of light in 1905[1]. The problem left by Poincarè in his book “Electricitè et optique”[2] was that a well-developed theory should be able to prove this [relativity] principle very strictly and all at once.

By the way what do you do when you prove that Euclidean geometry holds completely the same worlds on the front and back symmetric plane? This is the answer for this problem.

Proof

The oblique coordinate systems that face each other with the right hand system are placed on the front and back sides of a plane, and the origins are aligned. We define a 2x2 matrix B as a back surface coordinate transformation. If $B \neq B^{-1}$, then we are able to distinguish which side of a plane we are on. Because it is not distinctive which side of a plane is the back or front,

the symmetry plane* equation is $B = B^{-1} \Leftrightarrow B^2 = E$.

Since it is an inside out transformation, then $\det B < 0$.

*Symmetry plane means the front and back symmetric plane.

The solution is the oblique reflection transformation $B = \begin{pmatrix} -a & -b \\ kb & a \end{pmatrix}$, $\det B = a^2 + kb^2 = -1$.

The oblique reflection transformation B is mirror-inverted to derive the coordinate transformation F between the right hand systems on the same surface.

$$F = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} B = \begin{pmatrix} a & b \\ kb & a \end{pmatrix}, \quad \det F = 1, \ k \text{ is a commutative coefficient.}$$

When $k = -1$, the transformation F is referred to as a rotation transformation, when $k = 0$, the transformation F is referred to as a Galilean transformation, and when $k > 0$, the transformation F is referred to as a Lorentz transformation.

If k is fixed on a plane, it is proved [4] [5] that
when $k = -1$, this plane is made by space x space and it is completely isotropic,
when $k = 0$, this plane is made by space x absolute time and it is semi-isotropic,
and when $k > 0$, this plane is made by space x time and it is semi-isotropic.

The transformations B and F form a transformation group whose orbit is the quadratic invariant function $\phi(p)$, and have a transformation invariant r^2.

$$\phi(Bp) = \phi(Fp) = \phi(p) = -kx^2 + y^2 = r^2, \quad p = \begin{pmatrix} x \\ y \end{pmatrix}.$$

For the front and back symmetric plane, each of the isometric transformation whose length (norm) is $\| p \| = r$ owing to the difference of the sign of k forms an isometric transformation geometry.

Assuming that the back surface point q corresponds to the front surface point p of a plane, $q = Bp$ from the back coordinate transformability of the transformation B, and $p = Bq$ from the figure inversion transformability, therefore

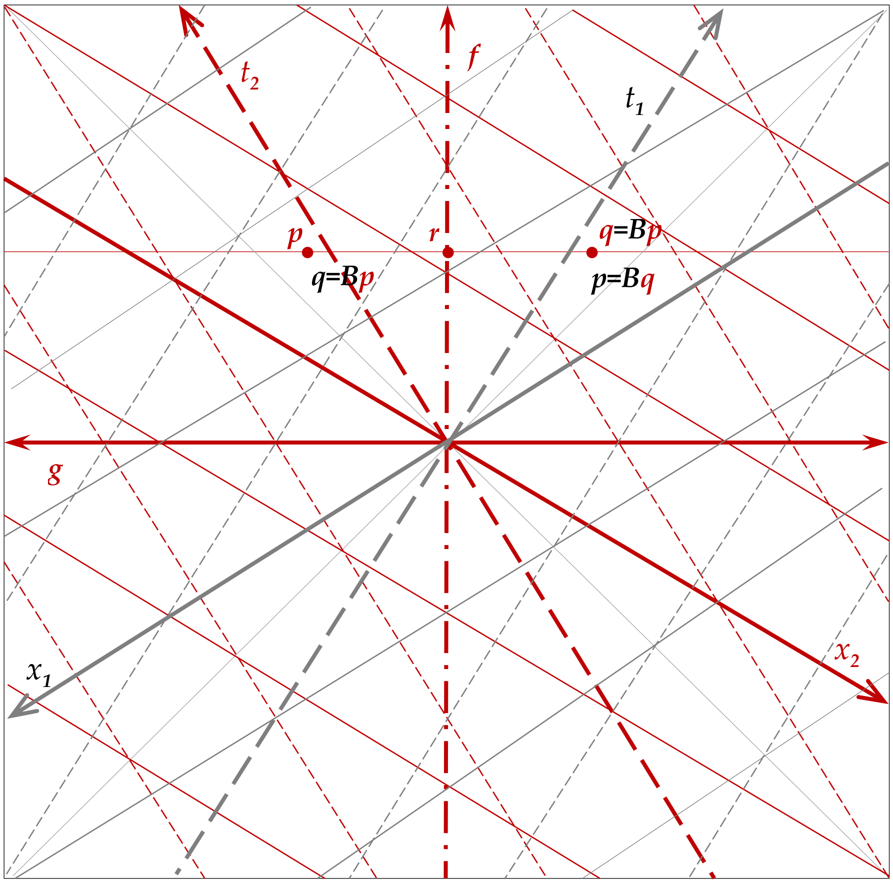
$$p = Bq = B(Bp) = p.$$

Transformation B guarantee that the symmetry plane holds exactly the same worlds of the front and back symmetric coordinate plane.

Two inertial coordinate systems (space-time two-dimensional model) that move at a relative constant velocity on a straight line are symmetrical to each other, and are in a relationship of front and back symmetric coordinate systems of a space x time plane (Minkowski plane).

This means the relativity principle in which any basic law of nature with vector $\binom{x}{y}$ or $\binom{x}{t}$ must be oblique reflection transformation B invariant, and as a result rotation transformation invariant or Lorentz transformation invariant, because $F=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}B$ and basic laws have symmetry of x inversion.

From the above, based on a linear plane,
from the front and back symmetry of the space x space plane, the reflection transformation group and the rotation transformation group form Euclidean geometry,
from the front and back symmetry of the space x absolute time plane, the oblique reflection transformation group and the Galilean transformation group form Newtonian mechanics,
and from the front and back symmetry of the space x time plane, the oblique reflection transformation group and the Lorentz transformation group form the relativity principle and the special relativity theory is established [3]. □



Oblique coordinate system (from back side view:red color)

Oblique reflection plane stretched by $B=\frac{1}{\sqrt{17}}\begin{pmatrix}-9 & 8\\ -8 & 9\end{pmatrix}$.

x: space axis
t: time axis
x_1-t_1 the front coordinate plane
x_2-t_2 the back coordinate plane

f : fold line
g : isotropic line

$q=Bp$ figure inversion transform.
$q=Bp$ back coordinate transform.

g // line p–q

Reference

[1] A. Einstein “On the electrodynamics of moving bodies” 1905

[2] H. Poincarè “Electricitè et optique” 1901

[3] H. Poincarè “Science and Hypothesis” 1902 “The reason why true proof produces various results is that the conclusion is in a sense more general than the premise. ”

[4] Hiroaki Fujimori, web site http://www.spatim.sakura.ne.jp/

[5] Hiroaki Fujimori, YouTube “Relativity Arises from the Symmetry of Spacetime” 13minutes