Matter versus Geometry

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The distinction between matter and spacetime geometry in classical general relativity is almost always taken to be unproblematic, usually, even if only implicitly, conceived of as determined by the existence of an associated stress-energy tensor for a given field or structure. I argue that this conception suffers ambiguities even in the purely classical case, due to difficulties raised by singularities, the equivalence principle, and “self-sourcing” of curvature. In black hole thermodynamics and semi-classical gravity more generally, the distinction suffers more blurring from the seeming intertransformability of curvature and matter in the phenomenon of Hawking radiation. In quantum gravity, in so far as one is able to say anything with confidence, the distinction seems to break down entirely even at the most abstract and generic level. This suggests that approaches to quantum gravity that have such a distinction built in at a fundamental level may be problematic. More speculatively, it may suggest that “matter” and “curvature/gravity” are just different manifestations of an underlying “unified” entity.

In classical general relativity, the most “obvious” way to try to distinguish pure gravitational geometry from matter is to advert to the difference between Ricci and Weyl curvature: there is matter present when and only when $R_{ab} \neq 0$, for then and only then does $T_{ab} \neq 0$. Thus, Weyl curvature, being independent of $T_{ab}$ at each point of spacetime, represents pure gravitational geometry. Unfortunately, this clean picture is ruined by the fact that the Bianchi identity for the Weyl tensor using the Lanczos tensor shows that it depends on derivatives of $T_{ab}$:

$$
\nabla_n C^{abc} = J_{abc} := \frac{1}{2} \nabla_{[b} R_{a]c} + \frac{1}{6} g_{c[a} \nabla_{b]} R
$$

The classical versions of the equivalence principle also suggest the blurring of the line between matter and geometry/gravity:

1. gravitational mass is equal to inertial mass—the property that determines geometry is the property that responds to geometry (is “guided by” it in motion); but geometry “can source itself”, as in non-trivial vacuum spacetimes, and geometry can be “guided by itself” as well, as in the characteristics of gravitational radiation following null geodesics, so there is no difference between gravity and matter at this level at all;

2. that there can be an “effective gravitational field” even when there is no matter present and a system is “only accelerating” suggests, again, that the difference between matter and geometry/gravity is to some degree—not “conventional”, but, perhaps better, only effectively determined (in the sense of effective field theory).

Consider another way to try to characterize matter in general relativity: matter fields are those local degrees of freedom characterized by invariance under symmetry groups beyond diffeomorphisms. When a singularity forms from gravitational collapse, however, then those degrees of freedom vanish, as made precise by the No Hair theorems. This seems to be a case of “matter

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turning into curvature/geometry”. In semi-classical gravity, Hawking radiation is “curvature turning into matter” in this sense: it’s an interaction between “geometrical degrees of freedom” and “material degrees of freedom” such that the former excite the latter in a way that lead to more of the latter and less of the former, in a sense one can make precise (energy/mass content).

In quantum gravity, abstractly and generally considered, a pure “matter particle-creation operator” does not seem to be a well defined notion, in so far as non-trivial curvature should attend the creation of any thing like standard particles. Correlatively, even anything like a “Weyl-curvature creation operator” will generically entail the creation of something like ordinary matter, in virtue of the relation of the gradient of the stress-energy tensor to the divergence of the Weyl tensor (the Lanczos tensor, as shown above). It does not seem that there could be an operator for the one without the other, so “observables” in the standard sense of quantum theory cannot disambiguately matter from geometry.