Unification of theories in Hilbert Space and in Minkowski Space: Remark on some fundamental theorems

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Extended Abstract.

The unification of quantum mechanics and relativity theory is often considered the fundamental problem of theoretical physics. Quantum mechanics is described in Hilbert space, while relativity theory is described in 4-dimensional Minkowski space. I will here investigate one angle under which the unification of both theories is often believed to be *impossible*. This angle will be seen to be quite abstract and fundamental: it is that of the theorems of Bell and Kochen-Specker. Both theorems stipulate that quantum mechanics cannot be derived from, and thus reconciled with, a deterministic theory described in Minkowski space and satisfying the principle of relativistic causality, often called the principle of 'locality'. My aim is to show that these theorems are much more limited than generally believed: it appears they only prohibit the reconciliation between quantum mechanics and a very specific class of local deterministic theories – certainly not all of them.

Quantum mechanics is a probabilistic theory elegantly described in Hilbert space; relativity theory is a deterministic theory elegantly described in Minkowski space. The problems of unification may thus, on the most elementary level, stem from the very different causal nature of both theories; this causal nature might well represent their most basic or abstract property. It therefore seems that addressing the long-standing problem of unification from this angle is worthwhile. Since the rise of the Copenhagen interpretation of quantum mechanics, culminating in the theorem of Bell, it is generally believed that quantum probabilities can under no circumstances be described by underlying deterministic processes, at least not if these are local in the relativistic sense.

This belief is strengthened by very recent publications in Nature and Physical Review Letters announcing the penultimate death of 'local realism' [1-3]. The authors of [1-3] have succeeded in performing quasi-ideal Bell experiments, strongly violating a Bell-type inequality. According to these authors 'realism' is 'the assumption that objects have physical properties independent of measurement' [1]; and locality is the above mentioned principle of relativistic causality: no influence can travel faster than light. So, at least one of these basic principles of physics should be given up. Now, relativity theory is *the* paradigm of a local realistic theory; so the above claim sounds awkward in the ears of a relativity theorist. What is meant, in some more detail, is that local deterministic theories (*à la* relativity theory)

underlying and explaining quantum mechanics (the quantum probabilities) cannot exist. Such sub-quantum theories are termed 'hidden-variable' theories (HVTs), and the HV are typically thought to be deterministic variables in Minkowski space (this case is the most interesting one for us; in principle the variables could be more general). In any case, if these authors are right, we should give up all hopes to derive quantum mechanics from a deterministic theory compatible with relativity theory.

The aim of this article is to show that the death announcement of local realism and of deterministic HVTs compatible with relativity theory [1-3], is premature [4]. More specifically, the experiments [1-3] certainly warrant the claim that an important and natural class of local HVTs has been refuted, but it will be argued here that a surprisingly wide class of local HVTs still survives. The HVTs that I consider exploit the 'freedom-of-choice' loophole, or in other words the possibility that 'measurement independence' (MI) may not be valid within these HVTs [4]. As is well known, all Bell-type inequalities assume that MI is valid. This condition of stochastic independence is defined as follows:

$$\rho(\lambda|a,b) = \rho(\lambda|a',b') \equiv \rho(\lambda) \text{ for all } (\lambda,a,b,a',b') \qquad \text{(MI)}. \qquad (1)$$

Here ρ is the conditional probability distribution of the hidden-variables λ; a and a' are values of the left analyzer angle in the Bell experiment; b and b' of the right analyzer angle. MI is usually deemed obvious because violating it would mean that the hidden variables λ depend on (a,b), which means by standard rules of probability calculus that the analyzer angles (a,b) depend on the variables λ. But (a,b) can be freely or randomly chosen in experiments – so how could these angles depend on the λ (variables which moreover determine the probabilities for the left and right outcomes) ? Ergo, MI must hold, unless one accepts a nonlocal or conspiratorial world, or so it is believed. But it will be shown here that MI may be violated in certain systems in a local and non-conspiratorial manner, i.e. in experiments in which Alice and Bob are perfectly free-willed.

[1] B. Hensen et al., 'Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometers', Nature **526**, 682 (2015)
[2] L. Shalm et al., 'Strong loophole-free test of local realism', Phys. Rev. Lett. **115**, 250402 (2015)
[3] M. Giustina et al., 'Significant-loophole-free test of Bell's theorem with entangled photons', Phys. Rev. Lett. **115**, 250401 (2015)
[4] L. Vervoort, 'No-go theorems face background-based theories for quantum mechanics', Found. Physics **45**, 1 (2015), doi: 10.1007/s10701-015-9973-7