THE MEANING OF SPACETIME SINGULARITIES

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In the following, I will discuss the geometric and physical interpretation of spacetime singularities occurring in general relativity (GR).

GR has two main problems: the prediction of *singularities* [4], and the problem of *quantization*. Despite these problems, the predictions of GR continue to be confirmed by experiment, culminating recently with the detection of gravitational waves resulting from the merging of two black holes [3].

Given the repeated experimental confirmation of the predictions of GR as compared to the alternative theories, we should consider more carefully what GR itself has to say about singularities. This motivated my research program to find natural formulations of GR in terms of variables that remain finite at singularities, *cf.* [8] and references therein. I will briefly review these results, and then discuss the geometric and physical interpretation of singularities and singular spacetimes.

The problem with singularities is that the metric becomes singular. This means that some of the metric tensor components g_{ab} or g^{ab} become infinite. This prevents the construction of the covariant derivative $\Gamma^a{}_{bc}$ (since $\Gamma^a{}_{bc}$ requires the inverse of the metric) and the Riemann curvature $R^a{}_{bcd}$. However, in [11] I show that differential geometry can be extended in a natural and invariant way to singular metrics g_{ab} which are smooth and become degenerate ($\det g = 0$). Only the lower covariant derivative (in terms of Γ_{abc}) and the lower form of the Riemann curvature R_{abcd} remain finite at such singularities, but this turned out to be enough to describe a large class of singularities, and to rewrite Einstein's equation in terms of quantities that remain finite, and still be equivalent with the original Einstein equation outside the singularities [11, 9]. This applies to FLRW and more general big bangs [14, 7]. Black hole singularities are apparently not of this type, but this is because the usual coordinates are themselves singular, similarly to the case of the event horizon, which was resolved by Eddington [1] and Finkelstein [2]. However, a similar method could be used to make the $r = 0$ singularity of black holes smooth, albeit degenerate [6, 5]. The mentioned methods developed for degenerate metrics could then be applied, and the Schwarzschild solution could be extended analytically beyond the singularity. Singularities turned out to be compatible with global hyperbolicity [6, 12], improving our understanding of the information during black hole evaporation. Not only that the singularities in GR turned out to be understandable in terms of finite quantities, but they are also accompanied by *dimensional reduction* effects, which are researched in the last years because they allow the removal of infinities in *perturbative quantum gravity* [10].

If Nature prefers to use the proposed variables and atlases, it has to do this not just as a trick to avoid the infinities at singularities, but for more fundamental geometric and physical reasons. In the following, I try to elucidate these reasons.

Spacetime has a *topological*, a *differential*, and a *(geo)metric structure*, built one in top of another. The more fundamental are the topological and the differential structures. The metric is a dynamical quantity, which depends on the stress-energy of matter. Being dynamical, there is nothing to stop it from becoming degenerate at some places, and this is why singularities appear. The fact that the metric is less fundamental than the manifold structure agrees with our mathematical

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understanding of differential geometry. However, physically, it is possible that the *causal structure* (representing the type of separation between spacetime events) is more fundamental than the differential structure. This view is supported by the fact that the topology of lightcones is not affected at the important big bang and black hole singularities, while their differential structure is affected [13]. Another question is related to the connection and the curvature. The connection specifies isometries between the tangent spaces at infinitesimally closed events. If the lower connection is more fundamental, it should also admit a geometric interpretation. The lower connection, rather than connecting the tangent spaces, connects the tangent space at an event with the cotangent space at an infinitesimally closed event in spacetime. Its non-commutativity is expressed by the lower Riemann curvature R_{abcd}, which may be more fundamental, if we think that this tensor and not $R^a{}_{bcd}$ exhibits the known symmetries at permutations of indices, the decomposition in the Weyl and Ricci curvatures, and the corresponding spinorial decomposition.

Regarding the physical content, the proposed replacement of Einstein's equation is

$$R_{ab}\,\mathrm{d}_{vol} - \frac{1}{2}g_{ab}R\,\mathrm{d}_{vol} + g_{ab}\Lambda\,\mathrm{d}_{vol} = \frac{8\pi G}{c^4}T_{ab}\,\mathrm{d}_{vol}, \tag{1}$$

which is clearly equivalent to Einstein's outside the singularities, where $\mathrm{d}_{vol} \neq 0$, but its terms remain finite at singularities at least in some important cases. Is $T_{ab}\,\mathrm{d}_{vol}$ more fundamental than T_{ab}? It should be, considering that what we integrate in order to obtain the mass or the momentum are the volume forms of the form $T_{ab}u^a u^b\,\mathrm{d}_{vol}$. This is clear for example if the stress-energy corresponds to a fluid, $T_{ab} = (\rho + p)u_a u_b + p g_{ab}$. One integrates the differential forms $\rho\,\mathrm{d}_{vol}$ and $p\,\mathrm{d}_{vol}$, and not the scalar quantities ρ and p, which are not invariant, depending on the coordinates. This is consistent with the fact that on differentiable manifolds mathematicians integrate volume forms, not scalar or tensor fields. Also, the curvature terms have a geometric interpretation in this form. In addition, the Lagrangian density is $R\,\mathrm{d}_{vol}$, and the corresponding equations are (1) rather than the usual Einstein equation, which are obtained by dividing by d_{vol}, which is prohibited when the metric is degenerate, because they lead to infinities. In the particular case of the FLRW spacetime, the quantities $\rho\,\mathrm{d}_{vol}$ and $p\,\mathrm{d}_{vol}$ remain finite in the Friedman equations.

The above considerations suggest that the quantities used in rephrasing the geometry and physics to work at singularities are at least as adequate as the standard ones, both from physical and from geometric points of view.

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