Unification, decoupling, and identification in the "Palatini formalism"

1 Introduction

This paper focuses on several philosophical (and historical) aspects of the "Palatini formalism" as a "metric-affine" approach to general relativity. It is argued here that the history and the conceptual development of this formalism (mainly in the first two decades after the wake of general relativity), as well as its more recent incarnations, illustrate nicely the interplay of several concepts in philosophy of science. First, the focus here is on the unificatory power of a formalism based on a two-stage approach: first the decoupling of two mathematical structures by taking them as independent variables, followed by a *partial* identification of these structures. Second, it is important to relate operations such as decoupling and identifications of mathematical structures to mathematical *explanations*. Last but not least, this approach complements and augments current discussions on the role and value of variational principles in physics, especially in general relativity. Overall this paper attempts to show how mathematical constructs and assumptions have a role in the ontology of general relativity and of some of its "extensions."

The "Palatini formalism" is related to the Lagrangian formulation of general relativity. When used in the context of the action with the Lagrangian density, the "Palatini action," similar to the Hilbert action, outputs Einstein field equations.[1] Although it is equivalent to the Hilbert approach for a large class of problems, the "Palatini formalism" is nevertheless a "metric-affine" approach, in which the Lagrangian is a scalar density of the invariants of the curvature built from *both* the metric g and the connection Γ. This is in stark contrast with the "pure metric" or "pure affine" approaches where these

[1]For elementary introductions see Faraoni and Capozziello (2011); Wald (1984)

quantities are not independent: in the metric approach, the one adopted by both Einstein and Hilbert in their original deductions of the field equations, the metric fixes the causal structure of spacetime *and* the relations of measurements of spacetime distances with clocks and rods.

The main aim of this paper is to go beyond the "Palatini formalism" and to emphasize the *distinct* roles in the ontology of spacetime of these two structures (we prefer to talk about mathematical 'structures,' rather than 'objects'): on one hand, there is the *metric* $g_{\mu\nu}$, together with its Levi-Civita connection $\left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\}$ and, on the other hand, the torsion-less connection $\Gamma^{\mu}_{\alpha\beta}$. In the "Palatini formalism," the two mathematical structures g and Γ are taken to be independent quantities, and both are fundamental. We call their independent variation in an action principle the " $g-\Gamma$ decoupling." Second, based on some physical constraints, one can identify *a posteriori* aspects of the two structures and impose, for example, that the $\Gamma^{\mu}_{\alpha\beta}$ is identified with the Levi-Civita connection $\left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\}$ of $g_{\mu\nu}$. This process of identification is analyzed in this paper as "unification by identification." This type of unification is discussed in other contexts (thermodynamics, the electromagnetic unification, the electroweak unification, etc.) in the philosophical literature. (Glymour 1980; Morrison 2000)

2 A convoluted history

Some historical remarks are in order here. This paper surveys work in general relativity in the period between early 1920s and 1941 by Palatini, Einstein, Weyl, Eddington, and some recent "Extended Theories of Gravity," approaches based on corrections, generalizations, and extensions to the main theory, as reviewed by Faraoni and Capozziello (2011).

Recall that the "Palatini formalism" is a misnomer: the method was anticipated by Weyl and Eddington in the early 1920s, but was explicitly used by Einstein in three papers written in 1923 without mentioning Palatini's name. (Albert Einstein 1923a; Albert Einstein 1923b; Albert Einstein 1923c) Einstein will wrongly attribute this method to Palatini for the first time in (A. Einstein 1941) and later in many other papers. The method used by Palatini in (Palatini 1919) differs radically from the "Palatini formalism." For several reasons, the attempts by Eddington and Weyl in the early 1920s won't qualify as a "Palatini formalism" either; historical details are available in (Ferraris, Francaviglia, and Reina 1982 and references therein; Ray 1975; Cattani 1993). We adopt tacitly the conclusion of several historical studies that the original paper of Palatini is conceptually rather different than what we call now the Palatini formalism.

Second, this paper is part and parcel of a philosophical and historical approach to variational principles in physics. The role of variational principles in physics, and especially in general relativity, is a convoluted topic. Einstein and Grossmann had used, or misused, variational principles in their *Entwurf* theory in the 1913-1916 period: the problem was the limited covariance of the action. Lorentz and separately Hilbert have corrected partially this problem and set the Hamilton principle at the heart of the deduction of the gravitational field equation in 1916. Hilbert carried out the variational method by assuming that the gravitational Lagrangian $\mathcal{L}_{\mathrm{G}}$ is linear in the curvature R: $\mathcal{L}_{\mathrm{G}} = \sqrt{\mathrm{g}}\mathrm{R}$ and that the affine connection equals the Christoffel symbols of the metric g: $\Gamma^{\mu}_{\alpha\beta} = \left\{ \begin{array}{c} \mu \\ \alpha\beta \end{array} \right\}$. Later, Einstein acknowledged the virtue of Hilbert, Lorentz, among others, in deriving the field equations from variational principles.

At a first sight, the "Palatino formalism" as exposed be Einstein (Albert Einstein 1923c) integrates well with Hilbert's original motivation for developing the field equation of general relativity: the unification of gravitation and electromagnetism, or more precisely the attempt to *geometrize* on a common ground gravitation and electromagnetism.

The last motivation of the Palatini formalism is more related to developments in the 1980s. The reasons to extend gravity are multiple, but two are germane in this context: adding higher order curvature invariants to the curvature tensor R, or/and adding scalar fields which are minimally, or non-minimally, coupled to the gravitational field, most preeminently, this is the case of string theory or Kaluza-Klein theories. Another framework in which this formalism is relevant is the Brand-Dicke theory because of the non-minimal coupling ω with the scalar φ.

3 Unification, coupling and decoupling

In direct relation to the Palatini formalism, this paper remarks that if one takes the metric $g_{\mu\nu}$ and the connection $\Gamma^{\mu}_{\alpha\beta}$ as independent quantities, one *decouples* the mathematical structure of the metric from the structure of the geodesic. As a direct consequence, the causal structure of the spacetime is decoupled from its geodesic structure. It is also worth noting than in this case the Principle of Equivalence and the Principle of Causality become independent as a result of the decoupling of the two mathematical structures. This is indicative that "this decoupling enriches the geometric structure of spacetime and generalizes the purely metric formalism." (Faraoni and Capozziello 2011, 68)

There are three more 'far-fetched' consequences of the present analysis. First, the decoupling-identification succession entails that we clarify the meanings of both the concept of geodesic structure and metric by decoupling them in a formalism and then use identification to stabilize their semantics. The two concepts may be co-extensive in physics, but they have different conceptual meanings that may illuminate new physical aspects of gravity beyond its classical formulation. The Palatini formalism shows ultimately that corrections, generalizations and extensions to gravity are most likely opening the doors to "metalinguistic" interpretations of general relativity, contra (Curiel 2009). Second, this is an intriguing case of underdetermination of two formalisms by data, enticing philosophically and historically. Both Hilbert-Einstein and the Palatini actions belong to a large class of possible forms which all fit the observational data available. Explanatory and unificatory virtues of theories then may weight in and in this case one can state, as argued above, the advantages of the Palatini account in accommodating non-linear corrections to the Lagrangian as well as non-minimal couplings with scalar fields. Third, as stated here, the Palatini case can illuminate important aspects of variational principles in physics, and in this case the most preeminent are the consequences for the ontology of spacetime: its dual nature and its coupling aspect.

4 References

Cattani, Carlo. 1993. "Levi-Civita's Influence on Palatini's Contribution to General Relativity." In *The Attraction of Gravitation: New Studies in the History of General Relativity*, edited by John Earman, Michel Janssen, and John D. Norton. Springer Science & Business Media.

Curiel, Erik. 2009. "General Relativity Needs No Interpretation." *Philosophy of Science* 76 (1): 44–72. doi:10.1086/599277.

Einstein, A. 1941. “Demonstration of the Non-Existence of Gravitational Fields with a Non-Vanishing Total Mass Free of Singularities.” *Univ. Nac. Tucumán. Revista A.* 2: 5–15.

Einstein, Albert. 1923a. “Bemerkung zu meiner Arbeit ‘Zur allgemeinen Relativitätstheorie.’” *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin)* 1923: 76–77.

———. 1923b. “Zur affinen Feldtheorie.” *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin)* 137: 137–40.

———. 1923c. “Zur Allgemeinen Relativitätstheorie.” *Sitzungsberichte Der Königlich Preußischen Akademie Der Wissenschaften (Berlin)* 137: 32–38.

Faraoni, Valerio, and Salvatore Capozziello, eds. 2011. *Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics*. Fundamental Theories of Physics. Springer.

Ferraris, M., M. Francaviglia, and C. Reina. 1982. “Variational Formulation of General Relativity from 1915 to 1925 ‘Palatini’s Method’ Discovered by Einstein in 1925.” *General Relativity and Gravitation* 14 (3): 243–54. doi:10.1007/BF00756060.

Glymour, Clark. 1980. “Explanations, Tests, Unity and Necessity.” *Noûs* 14 (1): 31–50. doi:10.2307/2214888.

Morrison, Margaret. 2000. *Unifying Scientific Theories: Physical Concepts and Mathematical Structures*. Cambridge University Press.

Palatini, Attilio. 1919. “Deduzione invariantiva delle equazioni gravitazionali dal principio di Hamilton.” *Rendiconti del Circolo Matematico di Palermo (1884-1940)* 43 (1): 203–12. doi:10.1007/BF03014670.

Ray, J. R. 1975. “Palatini Variational Principle.” *Il Nuovo Cimento B (1971-1996)* 25 (2): 706–10. doi:10.1007/BF02724746.

Sidharth, Burra G., Marisa Michelini, and Lorenzo Santi, eds. 2014. *Frontiers of Fundamental Physics and Physics Education Research*. Springer Proceedings in Physics.

Wald, Robert M. 1984. *General Relativity*. Chicago: University of Chicago Press.