Is Spacetime Inextendible?

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Abstract

Introduction: John Earman (1995) has argued: “Metaphysical considerations suggest that to be a serious candidate for describing actuality, a spacetime should be [inextendible]. For example, for the Creative Force to actualize a proper subpart of a larger spacetime would seem to be a violation of Leibniz's principles of sufficient reason and plenitude. If one adopts the image of spacetime as being generated or built up as time passes then the dynamical version of the principle of sufficient reason would ask why the Creative Force would stop building if it is possible to continue.” Here, I will articulate three concerns I have with the condition of spacetime inextendibility.

Preliminaries: Here we consider relativistic spacetimes (M, g) where M is a smooth, connected, Hausdorff manifold without boundary and g is a smooth, non-degenerate, pseudo-Riemannian metric of Lorentz signature defined on M. We say a spacetime (M, g) is *extendible* if (M, g) is isometric to a proper subset of some other spacetime (M', g'). If a spacetime is not extendible, it is *inextendible*. Let us say that a spacetime with a property $\mathcal{P}$ is a *$\mathcal{P}$-spacetime*. If an extension of a $\mathcal{P}$-spacetime has $\mathcal{P}$, it is a *$\mathcal{P}$-extension*. A $\mathcal{P}$-spacetime with no $\mathcal{P}$-extension is *$\mathcal{P}$-inextendible*.

Concern 1 (Conceptual): It would seem that an inextendible spacetime is a spacetime which is ”as large as it can be”. But, there is a problem with this interpretation; the definition of inextendibility is a function of the class of all “possible spacetimes”. And it is unclear what constitutes this class. Consider an example: the “bottom half” of Misner spacetime. It is globally hyperbolic and it counts as extendible in the preceding. But suppose a version of the strong cosmic censorship conjecture is correct and all “physically reasonable” spacetimes are globally hyperbolic. Then shouldn't the bottom half of Misner count as being “as large as it can be”? In other words, shouldn't it count as being “inextendible”? Because of examples like these, there is a temptation to revise the definition of inextendibility. But a revision is less urgent if one can show that for a number of “physically reasonable” properties $\mathcal{P}$, the following is true: (*) every $\mathcal{P}$-inextendible $\mathcal{P}$-spacetime is inextendible. I show that (*) is false when $\mathcal{P}$ is the property of satisfying the weak energy condition. This settles a question posed by Geroch (1970).

Concern 2 (Metaphysical): My second worry is with the background metaphysics. Recall the justification for inextendibility: “the principle of sufficient reason would ask why the Creative Force would stop building if it is possible to continue” (Earman 1995). An important mathematical theorem due to Geroch (1970) underpins such metaphysical views: every extendible spacetime has an inextendible extension. But consider statement (**): every $\mathcal{P}$-extendible $\mathcal{P}$-spacetime has a $\mathcal{P}$-inextendible $\mathcal{P}$-extension. One wonders: for which “physically reasonable” properties $\mathcal{P}$, is it the case that (**) is true? One can show that (**) is true when $\mathcal{P}$ is the property of being globally hyperbolic or the property of having no closed timelike curves. But I show that (**) is false when $\mathcal{P}$ is the property of having every inextendible timelike geodesic be past incomplete. (Presumably the “big bang” in our own universe renders this property physically reasonable.) Thus, it is not clear to me that Mother Nature always has the option of building spacetime to the point where it cannot be made “any larger”

Concern 3 (Epistemological): My third worry is epistemological: It seems one can never know through our observations that spacetime is inextendible. Consider that following definition (Malament 1977): a spacetime (M, g) is *observationally indistinguishable* from a spacetime (M', g') if for every point $p \in M$, there is a point $p' \in M'$ such that the timelike pasts $I^-(p)$ and $I^-(p')$ are isometric. The intended interpretation is the following: if a spacetime is observationally indistinguishable from a second spacetime, then any observer in the first spacetime does not have the epistemic resources to know whether she is in the first spacetime or the second. I show the following: every spacetime without closed timelike curves is observationally indistinguishable from some other (non-isometric) spacetime which is extendible. Thus, there is a sense in which every observer in almost every spacetime will be unable to rule out the possibility of living in an extendible universe. And not only do we currently fail to have observational evidence to support the view that our universe is inextendible, but observers 10,000 years from now will inherit the same predicament.

References

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