Quantum Time

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We give a consistent quantum description of time, based on Page and Wootters' and on Aharonov and Kaufherr's conditional probabilities mechanism, that overcomes the criticisms that were raised against similar previous proposals. In particular we show how the model allows to overcome Pauli's objections against a time operator and how it can reproduce the correct statistics of sequential measurements performed on a system at different times

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This abstract is a (slightly modified) excerpt of [1], also available as a preprint in [2].

Time in quantum mechanics appears as a classical parameter in the Schrödinger equation. Physically it represents the time shown by a "classical" clock in the laboratory. Even though this is acceptable for all practical purposes, it is important to be able to give a fully quantum description of time. Many such proposals have appeared in the literature (e.g. a, necessarily incomplete list includes papers such as [3–14]). Some of these have received criticisms, e.g. [9, 15–19]. One of proposals is the Page, Wootters, Aharonov, Kaufherr (PWAK) mechanism [7, 8] (see also [4, 20–23]) which considers "time" as a quantum degree of freedom by assigning to it a Hilbert space $\mathcal{H}_T$. The "flow" of time then consists simply in the correlation (entanglement) between this quantum degree of freedom and the rest of the system, a correlation present in a global, time-independent state $|\Psi\rangle\rangle$. An internal observer will see such state as describing normal time evolution: the familiar system state $|\psi(t)\rangle$ at time t arises by conditioning (via projection) the state $|\Psi\rangle\rangle$ to a time t (Fig. 1), it is a conditioned state. The PWAK mechanism was criticized in [9, 15] and a proposal that overcomes these criticisms [24, 25] used Rovelli's evolving constants of motion [5, 26] parametrized by an arbitrary parameter that is then averaged over to yield the correct propagators. Although the end result matches the quantum predictions [27], the averaging used there amounts to a statistical averaging which is typically reserved to unknown physical degrees of freedom rather than to parameters with no physical significance. (A different way of averaging over time was also presented in [28] to account for some fundamental decoherence mechanism.)

In our paper [1] we use a different strategy: we show that the same criticisms can be overcome by carefully formalizing measurements through the von Neumann prescription [29] (which we extend to generalized observables, POVMs). We show how this implies that all quantum predictions can be obtained by conditioning the global, timeless state $|\Psi\rangle\rangle$: this procedure gives the correct quantum propagators and the correct quantum statistic for measurements performed at different times, features that were absent in the original PWAK mechanism [9, 19]. We also show how the PWAK mechanism can be extended to give the time-independent Schrödinger equation and give a physical interpretation of the mechanism.

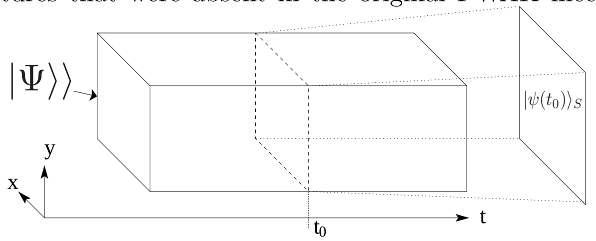

FIG. 1. Pictorial representation of the global state $|\Psi\rangle\rangle$. The Hilbert space of the system $\mathcal{H}_S$ is represented by the x, y axes, the time Hilbert space $\mathcal{H}_T$ by the horizontal axis. The state of the system $|\psi(t_0)\rangle$ at time t_0 of the conventional formulation of quantum mechanics (dashed lines) is obtained by conditioning $|\Psi\rangle\rangle$ to having time t_0.

What is the physical significance of the quantized time in the PWAK representation? One is free to consider the time quantum degree of freedom either as an abstract purification space without any physical significance or as a dynamical degree of freedom connected to some system, or collection of systems, that represents a clock that is used to define time. The latter point of view may describe an operational definition of time [30, 31] that is appropriate for proper time: it entails defining proper time as "what is read on a clock", where a clock is a specific physical system (described by the Hilbert space $\mathcal{H}_T$). In our paper we do not make commitment on any of these interpretations: our aim is only to elucidate some technical aspects of the representation and to clarify how it can be used to reproduce the predictions of standard quantum mechanics.

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