

Incongruent Counterparts and the Nature of Space

My paper provides a systematical investigation over the ontological nature of space. On the one hand we have *substantivalism*. On the other hand we have *relationalism*.

However, there is another line of argument that traces back to Kant, which is purely geometrical in nature – thus avoiding any reference to kinematics or dynamics.

In this first paragraph I will present my own reconstruction of the Kantian geometrical argument. In *On the First Ground of the Distinction of Regions of Space* Kant (1768) draws our attention to particular material bodies which resemble each other almost perfectly yet they cannot be made to be congruent by any means of a *simple rigid motion*. These objects are called “incongruent counterparts” and it is the existence of these objects which threatens the relationalist metaphysics. Two things (x,y) are said to be incongruent counterparts *iff*: (i) they cannot be made to be congruent by means of any continuous rigid motion and yet (ii) can be made to be congruent by means of a mirror reflection.

My reconstruction is the following:

1. Every object that has a possible incongruent counterpart has a definite handedness.
2. Every hand is a left hand or a right hand.
3. Any two objects (x,y) that are incongruent counterparts have different handedness.
4. There is a spatial difference between right and left hand.
5. According to relationalism, spatial difference should be accounted for in terms of different relations between objects.
6. So, according to relationalism, every spatial relation between object boils down to: i) spatial distances between objects; ii) spatial angles between objects.
7. According to relationalism, the difference in handedness should be accounted for in terms of: iii) spatial distances between parts of the hands; iv) spatial angles between those parts.
8. For any two arbitrary parts (a, b) of the left hand, such that $D(a, b)$ is the distance from a to b, and for any three arbitrary parts of the left hand (a, b, c) such that $A(a, b, c)$ is the angle between a-b and b-c, there are parts a^*, b^*, c^* of the right hand such that $D(a^*, b^*) = D(a, b)$ and $A(a^*, b^*, c^*) = A(a, b, c)$.
9. There is no difference in spatial relations between parts of the hands that could account for the difference in their handedness.

10. C1: Relationalism cannot provide a satisfactory account of the spatial difference of incongruent counterparts.

This C1 is a negative conclusion in that it is an argument *against* relationalism, and *not a direct* argument in favor of substantivalism. But we know that Kant wishes to go further, here is a possible way to do so:

11. There is “something” that grounds the difference in handedness.

12. There must be another entity besides the right and left hand such that this difference is explained in terms of the relations that those hands bear to that different entity.

13. (Universal) space provides the only ground to account for the difference in handedness.

14. C2: (Universal) space exists.

Conclusion C2 is stronger than C1 in that it offers a positive argument in favour of substantivalism.

In the third section I will provide two different arguments on behalf of the relationalist in order to counter both C1 and C2.

The first argument is a classic *Tu Quoque*: let us assume that there is something valid in conclusion C1. This conclusion has a compelling point only insofar as substantivalism has a better explanation for the difference in handedness (the existence of the universal independent space). But has she? I will give some reasons to be skeptical about this last claim: first, I will make the simplifying assumption that the substantivalist space is a collection of unextended spatial points and that every non-empty collection of points constitutes a spatial region. If such a space is admitted in one’s ontology, two different types of properties and relations are gained:

- i) properties or relations between spatial regions and
- ii) relations between spatial regions and material objects located at those regions.

According to analytic metaphysicians we know that: if a material object x , is exactly located at a spatial region R , x shares all the relevant geometrical properties with R . So, our intention is to analyze the spatial regions at which the left and the right hand are respectively located. This spells the ground for the *Tu Quoque* argument: for every relation between spatial regions there is a corresponding relationship between material parts of a hand. Either the argument in (1)-(10) is somewhat flawed both ways, in which it undermines *both* relationalism and substantivalism. This

seems to undermine C1.

A second argument in favour of relationalism is the one I called “*Ideology vs Ontology*”. Shortly: Premise (6) of the argument for C1 constructs relationalism rather poorly. It maintains that relationalism is committed to reducing all spatial relations to distances and angles. But why should it be so? If substantivalists are allowed to augment their *ontology* why should relationalist be denied to augment their own *ideology*? In particular they could add a new *primitive spatial determinable property*, namely handedness, which could be the explanation of the difference in handedness.

It seems fair to say that geometrical considerations alone do not suffice to vindicate one metaphysics of space over another. Probably, a quasi-geometrical argument in favor of substantivalism could be formulated considering the global topological notion of orientability.

This argument needs an independent scrutiny; however, it could be the case that orientability is not a purely geometrical property, yet it could be that orientability is inherited by some *law-like* features that describe the behavior of material objects.