On the Alleged Incommensurability of Newtonian and Relativistic Mass

One of the enduring debates about scientific change concerns the extent to which there is conceptual continuity across successive theories. The same term as used in different theories often on its face appears to have ultimately different extensions. Despite some ostensive overlap, the traditional story goes, they are embedded in a different network of terms that, holistically, grants it a different meaning. There has also been a more recent resurgence of debate regarding limiting-type relationships between theories, especially in physics, and whether these count as *reductive* relationships. This debate has concerned to what extent one theory can be the limit of another, and whether, if it is, this *explains* the limit theory. Although these two debates are not always explicitly connected, one of my goals is to show how a particular sort of positive solution to the reduction question can also contribute to understanding the extent of conceptual continuity and discontinuity between theories related by a limit. In particular, I apply some relatively new (to the philosophical literature) topological tools for understanding the limiting relationship between Newtonian and relativistic kinematics to what is perhaps the most well-known alleged example of conceptual incommensurability, that between the Newtonian and relativistic concepts of mass. My main contention is that the mass concept in the two theories of kinematics is essentially the same.

Famously, of course, both Kuhn and Feyerabend provided historical evidence that, in the mathematical framework used to formulate Newtonian and relativistic kinematics at the latter's inception in 1905, these concepts were not the same. I do not intend here to dispute their historical claims. Rather, my contention is based on a reconstruction of both theories in light of the best mathematical frameworks for describing them now, that of four-dimensional differential (affine) geometry. Thus, I do not intend to dispute here how historical actors involved in the construction, elaboration, and propagation of relativity theory. Instead, I wish to show that however the situation appeared to these actors, there is a way of describing and understanding these theories and their relationship that makes completely transparent the commonality of their concepts of mass.

One of the interesting conclusions to draw from this is that the usual understanding of incommensurability is likely too tied to the contingent and accidental features of the particular language in which a theory may be described—that is, it is too tied to the syntactic conception of theories that dominated philosophy of science in the 1960s. While there continues to be debate about the merits of the semantic view of theories, the syntactic view's successor, almost all seem to be in agreement that capturing the structure of a theory involves in large part aspects that are invariant (or at least appropriately covariant) across choice of language. Taking this into account shows that the essential differences are not so invariant. This moral is important for the reduction literature, too, for one potential objection to the claim that Newtonian kinematics is the *reductive* limit of relativistic kinematics is that the incommensurability of their mass concepts prevents the limit from being reductive, i.e., explanatory. Thus showing the commonality of the mass concepts is also important for understanding the explanatory relationship between the theories.

The technical portion of my argument proceeds in three phases. The first involves formulating both Newtonian and (special) relativistic kinematics in the framework of four-dimensional differential (affine) geometry, with the worldlines of particles as certain (timelike) piecewise smooth one-dimensional submanifolds. In both kinematical theories, mass is a non-negative parameter that, when associated with a worldline, specifies the degree to which the worldline departs from being a geodesics—following locally straight ("unforced") motion. The mass parameter then in both theories enters into the expression of the particle's four-momentum as a kind of normalization constant. I point out that there is a degree of convention not normally recognized in how it so enters, but that the choice of convention is essentially irrelevant when considering the details of simple particle collisions. The completion of this formulation reveals that mass plays the same functional roles in both kinematical theories; the only substantive difference lies in different spacetime structures that determine spatial distances and temporal lengths.

These different structures are nonetheless related, and in the second technical phase, I show how the Newtonian structure arises at the limit of the relativistic structure. This limit is constructed mathematically, by considering sequences of relativistic spacetimes (with various particles and observers within) that converge to Newtonian spacetimes, the sense of convergence being given by an appropriately chosen topology on the joint class of spacetimes. Because the Newtonian and relativistic spacetimes have a common conceptual interpretation, as revealed in the first phase, the topology can be easily interpreted as encoding similarity of empirical predictions. Thus a convergent sequence of relativistic spacetimes does not indicate a sequence in which the speed of light grows without bound, but rather one in which the measurements of the fixed observers can be better and better approximated by those of a certain hypothetical idealized Newtonian observer.

The third phase responds to a natural objection to the above account, namely that it has not explained the significant difference of Einstein's mass-energy relation, $E=mc^2$. Here I build on previous work by Rindler, Lange, and Flores, as well as on the conventional elements mentioned above, to explain the significance of the most famous equation not asserting the identity of mass and energy, but either as defining energy or stating an energy content associated with mass. The analysis of classical "fission" experiments can then be made where change in mass is interpreted only as a change in effective mass, a conceptual move also available in the Newtonian framework. Lastly, I gesture towards how this analysis extends to the Newtonian and general relativistic theories of gravitation, the former in its Newton-Cartan form, where the presence of the same sort of mass can be understood as having the same sort of influence on spacetime geometry.