Spontaneous symmetry breaking and the Unruh effect

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In this work we consider the ontological status of the Unruh effect. Is it just a formal mathematical result? Or the temperature detected by an accelerating observer can lead to real physical effects such as phase transitions? In order to clarify this issue we use the Thermalization Theorem to explore the possibility of having a restoration of the symmetry in a system with spontaneous symmetry breaking of an internal continuous symmetry as seen by an accelerating observer. We conclude that the Unruh effect is an ontic effect rather than an epistemic one, giving rise, in the particular example considered here, to a phase transition (symmetry restoration) in the region close to the accelerating observer horizon.

Trying to understand better Hawking radiation, Unruh did an amazing discovery in 1976. He realized that an observer moving through the Minkowski vacuum with a constant acceleration a will detect a thermal bath at temperature:

$$T = \frac{a\hbar}{2\pi c k_B}. \tag{1}$$

This result was first obtained for free bosonic quantum fields but later it was extended to interacting fields giving rise to the so called Thermalization Theorem. The relevance of the above formula is based, among other things, on the fact that it relates Quantum Mechanics, Relativity and Statistical Physics because it contains the Planck constant $\hbar$, the speed of light c and the Boltzmann constant k_B (in the following we will use natural units with $c = \hbar = k_B = 1$). In four dimensional Minkowski space M^4 we can introduce Cartesian inertial coordinates $X^\mu = (T, X, Y, Z)$ with metric:

$$ds^2 = dT^2 - dX^2 - dY^2 - dZ^2. \tag{2}$$

For the accelerating observer with acceleration directed in the X direction it is natural to introduce the comoving coordinates defined as: $T = e^{ax}\sinh(at)/a, X = e^{ax}\cosh(at)/a, Y = y$ and $Z = z$. These coordinates have event horizons corresponding to $X = -T$ and $X = T$ and therefore they cover only the so called right Rindler region R ($X >\mid T \mid$). The accelerating observer can only feel the Minkowski vacuum fluctuations inside R. However those fluctuations are entangled with the ones corresponding to the left Rindler region L ($X < -\mid T \mid$) as in the Einstein, Podolsky and Rosen setting. The result is that she sees the Minkowski vacuum as a mixed state described by a density matrix ρ_R which, according to the Thermalization Theorem, can be written in terms of the Rindler Hamiltonian $\hat{H}_R$ (the generator of the t time translations) as:

$$\hat{\rho}_R = \frac{e^{-2\pi \hat{H}_R/a}}{Tr e^{-2\pi \hat{H}_R/a}} \tag{3}$$

so that the expectation value of any operator $\hat{A}_R$ defined on the Hilbert space corresponding to R in the Minkowski vacuum $\mid \Omega_M >$ is given by:

$$< \Omega_M \mid \hat{A}_R \mid \Omega_M >= Tr \hat{\rho}_R \hat{A}_R. \tag{4}$$

This result can be seen as the one corresponding to a thermal ensemble at temperature $T = a/2\pi$ (in natural units) and it can be understood as a very precise formulation of the Unruh effect.

In any case one can of course wonder about the ontological status of this effect. Is the above result just formal or it truly represents a thermal effect? In Unruh's words: Could it be possible to cook a steak by accelerating it? More technically speaking: Can the Unruh effect give rise to phase transitions?

In order to explore this issue we have considered a model featuring a spontaneous symmetry breaking, namely the well known $SO(N+1)$ Linear Sigma Model (LSM). This model is defined by the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - V\left(\Phi^T\Phi\right) + J\sigma \tag{5}$$

where the multiplet $\Phi = (\bar{\pi}, \sigma)$ contains $N+1$ scalar fields ($\bar{\pi}$ is an N component scalar multiplet). The potential is given by:

$$V\left(\Phi^T\Phi\right) = -\mu^2 \Phi^T\Phi + \lambda\left(\Phi^T\Phi\right)^2 \tag{6}$$

where λ is positive in order to have a potential bounded from below and we consider μ^2 to be positive in order to provide a spontaneous symmetry breaking (SSB). When the external field is turned off ($J(x) = 0$), the SSB pattern is $SO(N+1) \to SO(N)$ and N Goldstone bosons appear in the spectrum.

At the tree level and $a = 0$ the low-energy dynamics is controlled by the broken phase:

$$< \Omega_M \mid \hat{\pi}^a \mid \Omega_M >= 0; \quad < \Omega_M \mid \hat{\sigma} \mid \Omega_M >= v. \tag{7}$$

where $v^2 = NF^2 = \mu^2/2\lambda$. Then the relevant degrees of freedom are the $\hat{\pi}$ fields which correspond to the Goldstone bosons (pions). Fluctuations along the σ direction correspond to the Higgs, the massive mode which is relevant at higher energies or temperatures.

By using the Thermalization Theorem it is also possible to obtain the Minkowski vacuum expectation value (VEV) of the squared σ field in the large N limit, which at $x = 0$ is given by:

$$< \Omega_M \mid (\hat{\sigma}(0))^2 \mid \Omega_M >= v^2 \left(1 - \frac{a^2}{a_c^2}\right) \tag{8}$$

for $0 \leq a \leq a_c$ and $< \Omega_M \mid (\hat{\sigma}(0))^2 \mid \Omega_M >= 0$ for $a > a_c$. Here the critical acceleration a_c is given by: $a_c^2 = 3(4\pi)^2 v^2/N$. This is exactly the thermal behavior of the LSM in the large N limit with a/a_c playing the role of T/T_c as seen by an inertial observer. It corresponds to a second order phase transition at the critical acceleration $a = a_c$ where the spontaneous broken symmetry for $a < a_c$ is restored for the accelerating observer for $a > a_c$. This result can be extended to any point of the R region the result being:

$$< \Omega_M \mid (\hat{\sigma}(x))^2 \mid \Omega_M >= v^2 \left(1 - \frac{a^2}{a_c^2} e^{-2ax}\right). \tag{9}$$

Therefore the σ field VEV seen by the accelerating (comoving) observer is position dependent. This is not strange since the proper acceleration along the X direction is breaking the Minkowski translation (and rotation) invariance. Now let us assume a belonging to the interval $0 < a < a_c$. Then the squared VEV will be a function on the coordinate x ranging from v^2 for $x = \infty$ to zero, which is reached at some negative x value given by:

$$x_c = -\frac{1}{2a} \log \frac{a_c^2}{a^2} < 0 \tag{10}$$

where the phase transition takes place. Notice that this locus $x = x_c$ is indeed a surface because the VEV is y and z (as well as t) independent. Now it is possible to write the VEV in terms of the inertial coordinates X and T:

$$< \Omega_M \mid (\hat{\sigma}(x))^2 \mid \Omega_M >= v^2 \left(1 - \frac{1}{a_c^2(X^2 - T^2)}\right). \tag{11}$$

It is very interesting to realize that this function does not depend on the acceleration a but only on the critical acceleration a_c and on v. In other words the VEV landscape depends only on the v parameter defining the LSM, but not on the acceleration of the comoving observer.

Therefore we have shown that a continuous spontaneously broken symmetry is restored for an accelerating observer. For her the VEV of the field depends on the position and it vanishes beyond a surface in the horizon direction.

We see this fact as a solid evidence in favour of the ontic character of the Unruh effect.

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