On the Nature of Spacetime: Special Relativity in Five Dimensions

Marcoen J.T.F. Cabbolet
Department of Philosophy, Free University of Brussels, Belgium

Einstein’s Special Relativity (SR) has been published over a century ago, and in the meantime it has become an untouchable corner stone of modern physics. But not only that: in the research towards a unitary theory that underlies the contemporary interaction theories, it is a theoretical test for every new theory that it has to reproduce SR as a limit case.

That being said, the purpose of this talk is twofold: (i) to introduce SR in a five-dimensional (5D) spacetime with a curled-up fifth dimension of Planck length size, and to show that this is empirically equivalent to ‘standard’ 4D SR; (ii) to show that this 5D SR emerges from the Elementary Process Theory (EPT), which has been introduced in [1, 2]. Thus speaking, while this 5D SR is a theory on its own that is distinct from other 5D accounts of SR as all particles move with the speed of light in four-dimensional space, it can also be viewed as the concrete physical meaning of the abstractly formulated EPT—in particular, the curled-up fifth dimension has a distinct interpretation in the framework of the EPT.

(i) 5D SR
First of all, a 5D IRF is a pseudo-Riemannian 5-manifold, rigorously defined in [3]. The set of all spatiotemporal positions is the set \( \mathcal{M} = \mathbb{R}^4 \times [0,1) \), with \((x^0, x^1, x^2, x^3, x^4) \in \mathcal{M}\) representing successively the time coordinate \(x^0\), the coordinates \(x^1, x^2, x^3\) in “regular” 3D space, and a fourth spatial coordinate \(x^4\); Planck units are presupposed, so Planck length and Planck time are scaled to 1. The unit interval \([0,1)\) forms a topological circle with circumference 1: the fourth spatial dimension is thus curled-up. A metric tensor field is defined on \(\mathcal{M}\) so that

\[
\eta^{(5)}(\vec{x}, \vec{y}) = -x^0 y^0 + x^1 y^1 + x^2 y^2 + x^3 y^3 + x^4 y^4
\]

for any tangent space \(T_p(\mathcal{M})\) and any vectors \(\vec{x}, \vec{y} \in T_p(\mathcal{M})\) with coordinates \(x^i, y^j\) with respect to a standard basis of \(T_p(\mathcal{M})\). A null line in a 5D IRF is then a curve \(\mathcal{C} \subset \mathcal{M}\) parameterized by \(u \in \mathbb{R}\) such that at every point \(X \in \mathcal{C}\) the tangent vector \(\vec{v}\) satisfies \(\eta^{(5)}(\vec{v}, \vec{v}) = 0\).

Presupposing linear motion only, 5D SR then consists of the following three postulates:

**Postulate 1.** There is no preferred 5D IRF.

**Postulate 2.** For any inertial observer \(O\) the 5D world line of any particle is a null line in the 5D IRF of \(O\).

**Postulate 3.** For any particle, the total distance traveled in the curled-up fifth dimension between any two events \(E_1\) and \(E_2\) on its 5D world line is observer-invariant.

A surprising theorem of 5D SR is then that for any inertial observer \(O\), all particles move with light speed in 4D space. This follows straight from Post. 2: at any point of a particle’s world line in any 5D IRF, its five-velocity \(\vec{v}^{(5)} = (v^1, v^2, v^3, v^4)\) has to satisfy \(\eta^{(5)}(\vec{v}^{(5)}, \vec{v}^{(5)}) = 0\) which leaves \((v^1)^2 + (v^2)^2 + (v^3)^2 + (v^4)^2 = 1\). Q.E.D.
Let a particle at a given point in the 5D IRF of an inertial observer $O$ have a five-velocity $(1,v^1,v^2,v^3,v^4)$; then the 3-speed of the particle is the number $v_3 = \sqrt{(v^1)^2 + (v^2)^2 + (v^3)^2}$.

With that definition, the following two theorems follow from 5D SR:

**Theorem 4.** The 3-speed of light, $c$, is a universal constant that has the same value 1 for any inertial observer $O$. □

**Theorem 5.** No experiment can determine the absolute 3-speed of an observer. □

These Ths. 4 and 5 are the postulates of the ‘standard’ theory of SR: if we suppress the fourth spatial coordinate, the predictions of 5D SR are thus the same as those of ‘standard’ SR—the two theories are thus empirically equivalent. A lemma is then that the total distance in the curled-up fifth dimension traveled by a particle between two events is identical to the invariant interval $\sqrt{\Delta s^2}$ of the corresponding spatiotemporal displacement vector in 4D Minkowski space.

(ii) 5D SR as the physical meaning of the EPT

The Elementary Process Theory (EPT) consists of seven well-formed formulas (wffs) together with a physical interpretation. This yields the picture that these seven wffs describe for integers $k$ and $n$ what happens in the $k^{th}$ process from the $n^{th}$ to the $(n+1)^{th}$ degree of evolution, which is a generic individual process that takes place at supersmall scale in the universe of the EPT: we thus have the parameter ‘degrees of evolution’ in the EPT. In [4] it has been shown by an analysis of the generic process that the invariance of the squared interval of SR emerges from the EPT if we view the degrees of evolution as an additional dimension that can be modeled by the set $\mathbb{R}$ together with an equivalent relation $\sim$, given by

$$x \sim y \iff x \equiv y (\mod 1)$$

where $x \sim y$ has to be interpreted as ‘$x$ and $y$ are physically the same point’. As there is a natural bijection $\phi : [x]_{\sim} \leftrightarrow x$ between the cells $[x]_{\sim}$ of the partitioning of $\mathbb{R}$ induced by $\sim$ and the elements $x \in [0, 1)$, 5D SR can be seen as a concrete mathematical model of aspects of the abstractly formulated EPT, which has to be understood as follows:

(i) the curled-up fifth dimension of $M$ is a model of the set of physically distinct points in the dimension of degrees of evolution;

(ii) the 5D world line of a massive particle is a continuous approximation model of the successive definite positions attained by a massive particle in the universe of the EPT. Massive particles (electrons, positrons, etc.) exhibit stepwise motion in the universe of the EPT: each step is an observer-independent unit displacement in degrees of evolution [4]; in $M$, this is a leap $X \rightarrow Y$ with $(X)^4 = (Y)^4 = 0$.

(iii) the 5D world line of massless particles in 5D SR is a dimensionless particle model of the motion of massless matter quanta in the universe of the EPT; for any position $X$ on any such world line, we have $(X)^4 = 0$.

5D SR can thus be viewed as a theory that is emergent in the framework of the EPT, and that describes the nature of spacetime in the universe of the EPT under special-relativistic conditions.

**References**


