On the Nature of Spacetime: Special Relativity in Five Dimensions

Marcoen J.T.F. Cabbolet

Department of Philosophy, Free University of Brussels, Belgium

Einstein's Special Relativity (SR) has been published over a century ago, and in the meantime it has become an untouchable corner stone of modern physics. But not only that: in the research towards a unitary theory that underlies the contemporary interaction theories, it is a theoretical test for every new theory that it has to reproduce SR as a limit case.

That being said, the purpose of this talk is twofold: (i) to introduce SR in a five-dimensional (5D) spacetime with a curled-up fifth dimension of Planck length size, and to show that this is empirically equivalent to 'standard' 4D SR; (ii) to show that this 5D SR emerges from the Elementary Process Theory (EPT), which has been introduced in [1, 2]. Thus speaking, while this 5D SR is a theory on its own that is distinct from other 5D accounts of SR as *all* particles move with the speed of light in four-dimensional space, it can also be viewed as the *concrete physical meaning* of the abstractly formulated EPT—in particular, the curled-up fifth dimension has a distinct interpretation in the framework of the EPT.

(i) 5D SR

First of all, a **5D IRF** is a pseudo-Riemannian 5-manifold, rigorously defined in [3]. The set of all spatiotemporal positions is the set $\mathcal{M} = \mathbb{R}^4 \times [0,1)$, with $(x^0, x^1, x^2, x^3, x^4) \in \mathcal{M}$ representing successively the time coordinate x^0, the coordinates x^1, x^2, x^3 in "regular" 3D space, and a fourth spatial coordinate x^4; Planck units are presupposed, so Planck length and Planck time are scaled to 1. The unit interval $[0,1)$ forms a topological circle with circumference 1: the fourth spatial dimension is thus curled-up. A metric tensor field is defined on $\mathcal{M}$ so that

$$\eta^{(5)}(\vec{x}, \vec{y}) = -x^0y^0 + x^1y^1 + x^2y^2 + x^3y^3 + x^4y^4 \tag{1}$$

for any tangent space $T_P(\mathcal{M})$ and any vectors $\vec{x}, \vec{y} \in T_P(\mathcal{M})$ with coordinates x^i, y^j with respect to a standard basis of $T_P(\mathcal{M})$. A **null line** in a 5D IRF is then a curve $\mathscr{C} \subset \mathcal{M}$ parameterized by $u \in \mathbb{R}$ such that at every point $X \in \mathscr{C}$ the tangent vector $\vec{v}$ satisfies $\eta^{(5)}(\vec{v}, \vec{v}) = 0$.

Presupposing linear motion only, 5D SR then consists of the following three postulates:

Postulate 1. *There is no preferred 5D IRF.* □

Postulate 2. *For any inertial observer $\mathcal{O}$ the 5D world line of any particle is a null line in the 5D IRF of $\mathcal{O}$.* □

Postulate 3. *For any particle, the total distance traveled in the curled-up fifth dimension between any two events $\mathcal{E}_1$ and $\mathcal{E}_2$ on its 5D world line is observer-invariant.* □

A surprising **theorem** of 5D SR is then that for any inertial observer $\mathcal{O}$, all particles move with light speed in 4D space. This follows straight from Post. 2: at any point of a particle's world line in any 5D IRF, its five-velocity $\vec{v}^{(5)} = (1, v^1, v^2, v^3, v^4)$ has to satisfy $\eta^{(5)}(\vec{v}^{(5)}, \vec{v}^{(5)}) = 0$ which leaves $(v^1)^2 + (v^2)^2 + (v^3)^2 + (v^4)^2 = 1$. Q.E.D.

Let a particle at a given point in the 5D IRF of an inertial observer $\mathcal{O}$ have a five-velocity $(1, v^1, v^2, v^3, v^4)$; then the **3-speed** of the particle is the number $v_{(3)} = \sqrt{(v^1)^2 + (v^2)^2 + (v^3)^2}$. With that definition, the following two theorems follow from 5D SR:

Theorem 4. *The 3-speed of light, c, is a universal constant that has the same vale 1 for any inertial observer $\mathcal{O}$.* □

Theorem 5. *No experiment can determine the absolute 3-speed of an observer.* □

These Ths. 4 and 5 are the postulates of the 'standard' theory of SR: if we suppress the fourth spatial coordinate, the predictions of 5D SR are thus the same as those of 'standard' SR—the two theories are thus empirically equivalent. A **lemma** is then that the total distance in the curled-up fifth dimension traveled by a particle between two events is identical to the invariant interval $\sqrt{\Delta s^2}$ of the corresponding spatiotemporal displacement vector in 4D Minkowski space.

(ii) 5D SR as the physical meaning of the EPT

The Elementary Process Theory (EPT) consists of seven well-formed formulas (wffs) together with a physical interpretation. This yields the picture that these seven wffs describe for integers k and n what happens in the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution, which is a *generic individual process* that takes place at supersmall scale in the universe of the EPT: we thus have the parameter 'degrees of evolution' in the EPT. In [4] it has been shown by an analysis of the generic process that the invariance of the squared interval of SR *emerges from* the EPT if we view the degrees of evolution as an additional dimension that can be modeled by the set $\mathbb{R}$ together with an equivalent relation $\sim$, given by

$$x \sim y \Leftrightarrow x \equiv y (\text{mod } 1) \tag{2}$$

where $x \sim y$ has to be interpreted as 'x and y are *physically* the same point'. As there is a natural bijection $\phi : [x]_\sim \leftrightarrow x$ between the cells $[x]_\sim$ of the partitioning of $\mathbb{R}$ induced by $\sim$ and the elements $x \in [0, 1)$, 5D SR can be seen as a **concrete mathematical model** of aspects of the abstractly formulated EPT, which has to be understood as follows:

(i) the curled-up fifth dimension of $\mathcal{M}$ is a **model of the set of physically distinct points** in the dimension of degrees of evolution;

(ii) the 5D world line of a massive particle is a **continuous approximation model** of the successive definite positions attained by a massive particle in the universe of the EPT. Massive particles (electrons, positrons, etc.) exhibit stepwise motion in the universe of the EPT: each step is an observer-independent unit displacement in degrees of evolution [4]; in $\mathcal{M}$, this is a leap $X \to Y$ with $(X)^4 = (Y)^4 = 0$.

(iii) the 5D world line of massless particles in 5D SR is a **dimensionless particle model** of the motion of massless matter quanta in the universe of the EPT; for any position X on any such world line, we have $(X)^4 = 0$.

5D SR can thus be viewed as a theory that is *emergent* in the framework of the EPT, and that describes the nature of spacetime in the universe of the EPT under special-relativistic conditions.

References

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