▸ THOMAS BENDA, *An abstract ontology of relativistic spacetime.*
Institute of Philsophy of Mind, National Yang Ming University, Taipei, Taiwan.
E-mail: `tbenda@ym.edu.tw`.

Relativistic spacetime is usually characterized as a pair $\langle M, g\rangle$ in which both components, a manifold and a metric with Lorentz signature are viewed as a given. Various metrics provide for interesting models, which have been studied extensively.

To achieve precision as well as ontological clarity and to make various approaches comparable, an axiomatization of setting up spacetime is desirable.

The axiomatization proposed here ([1], [2]) postulates a primacy of worldlines as compared to spacetime points, emphasizing the role of causality in spacetime. It starts out from a basic level, the level of first-order logic. A formal theory ST is set up, which is a conservative extension of Zermelo's set theory with urelemente ZU. Its vocabulary, as compared to ZU, contains an additional, ternary predicate constant $\nmid$, read "intersects with __ at". There are 15 proper axioms, which are mostly formulated in the primitive language of ST. The only higher-level expressions in the proper axioms denote real numbers, smooth functions and first derivatives.

The semantics of ST is Tarskian as usual. The universe of ST is an extended set-theoretical hierarchy, whose base is broadened by urelemente. All its elements which are generated from the empty set alone, "pure sets", are interpreted as mathematical entities, the rest, "impure ests", as physical entities. Urelemente are interpreted as worldlines, so that worldlines are structureless primitive objects, Spacetime points are sets of worldlines.

By the axioms, an Alexandroff topology and, subsequently, a manifold are set up. Sets of spacetime points which each contain precisely one common worldline form curves in spacetime, called "worldline curves". Now standard definitions of differential geometry are applicable, such as of tangent vectors and connections. There is one connection which makes all worldline curves geodesics, the "geodesic connection". From the geodesic connection, a metric is constructed.

Causality is central to the construction and introduced at a low level, The set of spacetime point transformations that are isomorphic under the element relation are precisely those that preserve causality, that is, under an intended standard interpretation, those that preserve particle paths and thus kinematics.

Hence it is possible to obtain relativistic spacetime from primitive concepts which do not incorporate prima facie physical concepts, such as bodies or observers. The set-up rests, besides mathematical entities, on no more than intersecting worldlines.

The theory ST expresses a common ontology of mathematical and (some) physical entities. That there is something to be expressed presupposes a platonistic view of mathematical objects, specifically, taking serious the set-theoretical hierarchy as existing, since mathematics (except geometry) is reducible to set theory. The set-theoretical hierarchy $\mathcal{V}$ is arguably a structure that is ontologically prior to its elements. All its elements obtain their identity by the element relation, including its starting point, the nothing. For building up $\mathcal{V}$, only two principles are needed:
(S1) every set is uniquely determined by its elements and is individuated if its elements are individuated; and
(S2) every subset y of any set x is individuated and contains only individuated elements if x does so.
(S1) allows to objectify the nothing to the empty set. $\mathcal{V}$ is then constructed by executing a self-referential imperative. The construction has to be by imperative since V cannot be completely described in a non-circular way. All sets in $\mathcal{V}$ come about only by the element relation and have no intrinsic properties.

That view is applied to worldlines in conjunction with the (absolute) nothing on a basic level. The same construction by imperative leads to higher-level sets. The proper axioms establish an additional structure, of pairs of intersecting worldlines plus one marker of a linear location, so that ultimately spacetime is created.

Thus spacetime has been established as a structure which is based on a set of featureless individuals, the urelemente, which are interpretetd as worldlines. That picture is complete, insofar as all entities of spacetime, including worldlines, are understood as individuals. However, worldlines have a motive beyond individuality: they are thought of as true, formally inexpressible continua (unlike, say, the real numbers). When worldlines intersect, the continuity is broken, but still reflected in the value of the third place of the predicate constant $\nmid$, which marks intersection "at __".

It is tempting to push the interpretation of the theory ST further. Worldlines—urelemente—correspond to continuously flowing time. Space, in contrast, is made of points which are identical or distinct. Time is primitive to space, space is ontologically secondary to time.

Spacetime as constructed by the theory ST has no matter component and so the question of having it being subordinate to distributed matter in the sense of Leibniz is—at this stage—answered in a trivial way. Does that make spacetime a substance? It seems, we have to qualify and clarify that. Spacetime as a whole, being a structure that is ontologically prior to its members, exists by virtue of itself. But it would make no sense to speak, as of common substances, of spacetime as persisting through changes. Spacetime is rather Parmenidean, immutable, even given the contingency of the structure defined by the intersection predicate. Spacetime points are not substances since they ontologically depend on relations, just as pure sets in $\mathcal{V}$. Yet worldlines may well qualify as substances, but we have to keep in mind that ST describes them only insofar as they are individuals and undergo relations.

[1] Benda, Th., A formal construction of the spacetime manifold. ***Journal of Philosophical Logic***, 37 (2008), pp. 441-478.

[2] ——— An axiomatic foundation of relativistic spacetime, ***Synthese***, 192 (2015), pp. 2009-2024.