The capacitor acted as a velocity selector in Bucherer's experiment, selecting β-ray electrons of velocities given by equation (2.9). In Bucherer's experiment there was no electric field outside the capacitor plates, and after emerging from between the plates of the capacitor, the electrons moved in circular orbits in the magnetic field, before striking the photographic plate in fig. 2.2. From equation (2.7), if the deflection of the electrons is d, as shown in fig. 2.2, we have:

$$\frac{mu}{Be}=\frac{(D^2+d^2)}{2d}.$$

From equation (2.9), $u=E/B$, so that after rearranging we have:

$$\frac{e}{m}=\frac{2d}{(D^2+d^2)}\frac{E}{B^2}.$$

In S.I. units e/m is in coulomb per kilogramme (C kg^{-1}). By measuring d, D, E and B, e/m can be calculated. Some of Bucherer's results are given in Table 2.1.

u/c	e/m	$\frac{e}{m_0}=\frac{e}{m\sqrt{(1-u^2/c^2)}}$
0·3173	$1{\cdot}661\times10^{11}$ C kg^{-1}	$1{\cdot}752\times10^{11}$ C kg^{-1}
0·3787	$1{\cdot}630\times10^{11}$	$1{\cdot}761\times10^{11}$
0·4281	$1{\cdot}590\times10^{11}$	$1{\cdot}760\times10^{11}$
0·5154	$1{\cdot}511\times10^{11}$	$1{\cdot}763\times10^{11}$
0·6870	$1{\cdot}283\times10^{11}$	$1{\cdot}767\times10^{11}$

Table 2.1.

It can be seen that the experimental values of e/m depend on the speeds of the electrons. However, if one assumes that

$$m=\frac{m_0}{\sqrt{(1-u^2/c^2)}},\tag{2.10}$$

where u is the speed of the β-ray electron and c is the speed of light, and if one calculates

$$\frac{e}{m_0}=\frac{e}{m\sqrt{(1-u^2/c^2)}},$$

then the calculated values of e/m_0 given in Table 2.1 are remarkably constant. They are as good a set of results as the reader is ever likely to obtain in his own laboratory work. In the spirit in which physical laws are 'established by experiment' in elementary practical courses, we will conclude from Bucherer's *experiment* that equation (2.10) is established *by experiment*. The quantity m in equation (2.10), which appears also in equations (2.3), (2.4), (2.5), (2.6) and (2.7), is generally called the *relativistic mass* or just the *mass* of the particle. The quantity

m_0, which is the value of m when $u=0$, is called the *rest mass* or *proper mass* of the particle. It can be seen that as the velocity of the particle increases, according to equation (2.10) the mass of the particle increases.

Notice we assumed that the charge $-e$ on the electron was independent of its velocity. Instead of saying that mass varied according to equation (2.10), we might be tempted to say that the charge q on a particle varied according to the equation:

$$q=q_0\sqrt{(1-u^2/c^2)},\tag{2.11}$$

where u was the velocity of the charge, q_0 the value of the charge when it was at rest, and that the mass m was invariant. Such assumptions would account for the results given in Table 2.1. There is, however, independent evidence in favour of the principle of constant electric charge. For example, if the charge on a particle did vary with velocity according to equation (2.11), then hydrogen atoms and molecules would not be electrically neutral, since the negative electrons are moving in orbits around the atomic nuclei in hydrogen atoms and molecules, and on average are moving faster than the positive nuclei (protons in this case) relative to the laboratory. If the charge did vary with velocity, hydrogen molecules should be deflected in electric fields, e.g. of the type shown in fig. 2.1 *a*. In 1960 King showed that the charges on the electrons and the protons in hydrogen molecules were numerically equal to within one part in 10^{20}. We therefore conclude that the charge on a particle is independent of its velocity and that the mass of a particle varies with the particle's velocity, according to equation (2.10).

To simplify the mathematics, we will sometimes make the trigonometrical substitution:

$$u=c\sin\theta \text{ or } u/c=\sin\theta,\tag{2.12}$$

where u is the velocity of the particle and c is the velocity of light. Substituting in equation (2.10):

$$m=\frac{m_0}{\sqrt{(1-\sin^2\theta)}}=\frac{m_0}{\sqrt{(\cos^2\theta)}}=m_0\sec\theta.\tag{2.13}$$

The variation of mass with velocity can be shown by plotting $m/m_0=\sec\theta$ against $u/c=\sin\theta$ as shown in fig. 2.3. As $u\to c$, m/m_0 tends to infinity. For normal laboratory speeds the variation of mass with velocity is negligible. Consider a train going at 100 kilometre per hour, which corresponds to $u/c\simeq10^{-7}$. In this case:

$$m=\frac{m_0}{\sqrt{(1-10^{-14})}}=m_0(1-10^{-14})^{-1/2}.$$

According to the binomial theorem, if $x\ll1$:

$$(1+x)^n\simeq1+nx.$$